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Research Article

Population aging caused by a rise in the sex ratio at birth

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Zhen Zhang¹

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Abstract

BACKGROUND

In the last several decades, notwithstanding its historical and biological stability, the sex ratio at birth (SRB) has risen in various parts of the world. The resultant demographic consequences are well documented and typically include ‘missing girls/women’ and the ‘marriage squeeze.’ However, the underlying mechanism and impact of the SRB on demographic dynamics have not been explored in depth.

OBJECTIVE

In this study, we investigate the impact of a rise in the SRB on the size, structure, and growth of a population, with a focus on population aging.

METHODS

We develop a simple methodological framework derived from classical stable population models to analyze how a rise in the SRB reduces population size and ages a population.

RESULTS

Cohorts born with a higher SRB are smaller than those born with a lower SRB. The smaller size of these cohorts leads to a reduction in the total population size, thereby increasing the proportion of cohorts of older persons born during periods with a lower SRB that are the same size as earlier cohorts. As cohorts continue to be born during the period with the higher SRB, their proportion in the population increases and the process of population aging accelerates.

CONTRIBUTIONS

This study shows that, in addition to fertility and mortality, the SRB can be a driving factor in population dynamics, especially when it rises well above normal biological levels.

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1. Introduction

A sex ratio at birth (SRB) of 1.05 male births to 1 female birth is considered biologically stable (Chahnazarian 1988). Since the late 1970s a rising trend in the SRB in some Asian and Eastern European countries has been evident (Babiarz et al. 2019; Cai and Lavelly 2003; Chao et al. 2019; Guilmoto 2012a; Guilmoto and Ren 2011). Reasons for this trend include prenatal sex selection driven by a rapid decline in fertility, a strong preference for sons, and easy access to sex-selection technologies (Guilmoto 2009, 2012a, 2012b, 2012c; Kashyap and Villavicencio 2016; Meslé, Vallin, and Badurashvili 2007). In light of mounting concern regarding the rapid rise in the SRB, the governments of various countries have introduced national policies and programs to address gender inequality and sex selection at birth, thereby lowering abnormally high sex ratios (Guilmoto 2012a; Li 2007). In recent years, South Korea has succeeded in reducing its SRB to a biologically normal level (Den Boer and Hudson 2017), and in other countries such as China the pace of the increase in the SRB has been reduced (Chao et al. 2019).

The demographic consequences of a high SRB, which typically include the phenomena of ‘missing girls/women’ and the ‘marriage squeeze’, have been well documented (Guilmoto 2012a, 2015; Jiang, Feldman, and Li 2014; Li 2007; Sen 1990; Tuljapurkar, Li, and Feldman 1995). Moreover, simulations conducted in several studies have shown that a higher SRB is often associated with population aging or a reduced pace of population growth (Cai and Lavelly 2003; Chen and Li 2010; Chen and Zhang 2019). However, there have been few in-depth explorations of the underlying mechanism driving these phenomena. Therefore, questions of whether the link between the sex ratio and population aging is coincident under specific population conditions, and if not, how it arises, and what factors are involved in the process, remain unanswered.

We aim to investigate the impact of an increase in male births on the population structure using a simple methodological framework derived from classical stable population models (Lotka 1939). Applying this framework, we perform a perturbation analysis to examine how a rise in the SRB reduces the population size and induces population aging.

The rest of this paper is organized as follows. First, we introduce the settings and notation used in this study. Second, we present a simple case in which the number of births remains constant over time. We formulate the relationship between the SRB and population size and visualize the process of population aging resulting from a rise in the SRB. Third, we present a case with constant fertility schedules, which is not only more realistic but also can show the impacts attributed solely to a rise in the SRB. Last, we discuss the implications of our findings.

2. Settings and notations

We based our analysis on an assumption of female dominance; i.e., population dynamics are determined by female vital rates and the number of males is always sufficient for pairs to be formed with all of the females. This assumption can be justified by the widespread phenomenon of excess males within all populations that demonstrate an abnormally high SRB (Guilmoto 2012a, 2012b, 2015; Sen 1990; Tuljapurkar, Li, and Feldman 1995). Additionally, net migration rates are assumed to be zero at all ages, so in effect the population is assumed to be closed to migration. We formulated the following notations to simplify our analysis:

- $SRB(t)$: the sex ratio at birth at time t expressed by the number of male births per female birth.
- $\pi_f(t)$ and $\pi_m(t)$: the proportions of female and male births at time t , with $\pi_f(t) + \pi_m(t) = 1$. The two proportions are equivalent to the SRB; for example, $\pi_m(t) = SRB(t)/(1 + SRB(t))$ or $SRB(t) = \pi_m(t)/(1 - \pi_m(t))$.
- $N_f(a, t)$, $N_m(a, t)$, and $N(a, t)$: female, male, and total populations aged a years at time t .
- $B_f(t)$, $B_m(t)$, and $B(t)$: female, male, and total births at time t . The numbers of female and male births are expressed as $B_s(t) = \pi_s B(t)$, where $s = f, m$, respectively. If the number of annual births is fixed, then $B(t) = B$.
- ω : the upper limit of the human lifespan.
- $p_f(a)$ and $p_m(a)$: the constant probability that a newborn female (male) baby will survive up to age a .
- $p(a)$: the constant probability that a newborn child will survive up to age a , which is calculated as the weighted average of $p_f(a)$ and $p_m(a)$: $p(a) = \pi_m \cdot p_m(a) + (1 - \pi_m) \cdot p_f(a)$.
- e_f^o , e_m^o , and e^o : life expectancy at birth for females, males, and the total population respectively, defined by the integral of $p(a)$ from age 0 to ω . For instance, $e^o = \int_0^\omega p(a) da$. The three life expectancies are related to one another, as indicated by $e^o = \pi_m \cdot e_m^o + (1 - \pi_m) \cdot e_f^o$, usually with $e_m^o < e_f^o$.
- $\delta(a) = p_f(a) - p_m(a)$: the sex difference relating to the probability of survival from birth to age a , which is usually positive because of the female survival advantage. Thus, $\int_0^\omega \delta(a) da = e_f^o - e_m^o$, which is the sex difference in life expectancy at birth.

- $m(a)$: the constant fertility schedules of women for their female and male offspring, combined with the total fertility rates; that is, $TFR = \int_{\alpha}^{\beta} m(a)da$, where α and β are the lower and upper bounds of the reproductive age span, respectively.
- $m_f(a)$ and $m_m(a)$: the constant fertility schedules for female and male offspring, respectively.

3. SRB-induced reduction in population size

In this section we demonstrate how a rise in the SRB can reduce the size of a population. Consider a stationary, female-dominant population that is characterized by a fixed number of annual births, a fixed life table for each sex, and zero net migration rates for all age groups. Accordingly, the number of persons aged a years at time t is equal to the number of births $t - a$ years earlier multiplied by the probability of surviving from birth to age a :

$$N(a, t) = B(t - a) \cdot p(a) = B \cdot p(a). \quad (1)$$

The total population size is thus expressed as:

$$N(t) = \int_0^{\omega} N(a, t)da = B \cdot \int_0^{\omega} p(a)da = B \cdot e^o, \quad (2)$$

where e^o denotes life expectancy at birth for the population as a whole.

For a two-sex population, $N(a, t)$ can be expressed in terms of the proportion of male births denoted by π_m :

$$\begin{aligned} N(a, t) &= B \cdot \left(\pi_m \cdot p_m(a) + (1 - \pi_m) \cdot p_f(a) \right) \\ &= \pi_m \cdot B \cdot \left(p_m(a) - p_f(a) \right) + B \cdot p_f(a) \\ &= -\pi_m \cdot B \cdot \delta(a) + B \cdot p_f(a). \end{aligned} \quad (3)$$

The derivative of Equation (3) with respect to π_m is:

$$\frac{dN(a,t)}{d\pi_m} = -B \cdot \delta(a). \quad (4)$$

The slope at age a , with the sign reversed, is the product of B and $\delta(a)$. This expression indicates how many more extra male births resulting from a rise in π_m would be lost up to age a because of the male survival disadvantage.

Analogously, the total population can be expressed as:

$$N(t) = B \cdot e^o = B \cdot (\pi_m \cdot e_m^o + \pi_f \cdot e_f^o) = -B \cdot \pi_m \cdot (e_f^o - e_m^o) + e_f^o. \quad (5)$$

The differentiation of Equation (5) produces the following equation:

$$\frac{dN(t)}{d\pi_m} = -B \cdot (e_f^o - e_m^o). \quad (6)$$

Because $e_f^o > e_m^o$ the equation result is negative, indicating the total number of extra deaths that would occur if more boys were born because of a rise in π_m .

The mechanism behind the above-described impact is intuitively straightforward. Given the number of births, the SRB is higher, and more male than female infants are born. Because male mortality rates are usually higher than female mortality rates, a higher number of male births than female births implies an increase in the number of deaths, leading to a reduction in the population size. As shown in Equation (5), when B (the number of births) is fixed and π_m is rising, the increasing weight of the male population will lead to decreasing e^o and hence to a reduced population size.

The impact of the SRB is symmetrical. Thus, a decrease in π_m or, equivalently, a rise in π_f can lead to an increase in the population size unless the assumption of female dominance, and vice versa, is invalidated by the resulting outcomes. The results of our analysis regarding the impact of changes in the SRB on the population size and structure can work in both directions relating to population size, as evidenced in Equations (4) and (6).

Dividing Equation (4) by Equation (1) yields the rate of the relative change in $N(a, t)$ given a rise in π_m :

$$\frac{d \ln N(a, t)}{d\pi_m} = \frac{dN(a, t)}{N(a, t) d\pi_m} = -\frac{B \cdot \delta(a)}{B \cdot p(a)} = -\frac{\delta(a)}{p(a)}. \quad (7)$$

Equation (7) shows that with the removal of the impact of the number of births, the resultant reduction in population size depends on the ratio of sex difference in the probability of survival up to age a to the same probability for the population as a whole. In the continuous life tables the probability of survival up to age a can also be

interpreted as the number of survivors at age a . Thus, $\delta(a)/p(a)$ indicates the ratio of the relative importance of the sex difference concerning the number of survivors at age a to the combined number of survivors of both sexes at the same age. A finding that a small $\delta(a)$ together with an even lower $p(a)$ could yield a high ratio is not surprising.

Analogously, dividing Equation (6) by Equation (2) gives the rate of the relative change in the total population as a response to a rise in π_m :

$$\frac{d \ln N(t)}{d\pi_m} = -\frac{e_f^0 - e_m^0}{e^0}, \quad (8)$$

meaning that the relative change in the total population is equal to the ratio of the sex difference in life expectancy at birth to life expectancy at birth for both sexes combined.

More generally, the subpopulation above a certain age is given by:

$$N_{x+}(t) = \int_x^\omega N(a, t) da.$$

The relative change in $N_{x+}(t)$ with respect to π_m is expressed as:

$$\frac{d \ln N_{x+}(t)}{d\pi_m} = -\frac{\int_x^\omega \delta(a) da}{\int_x^\omega p(a) da} = -R_x, \quad (9)$$

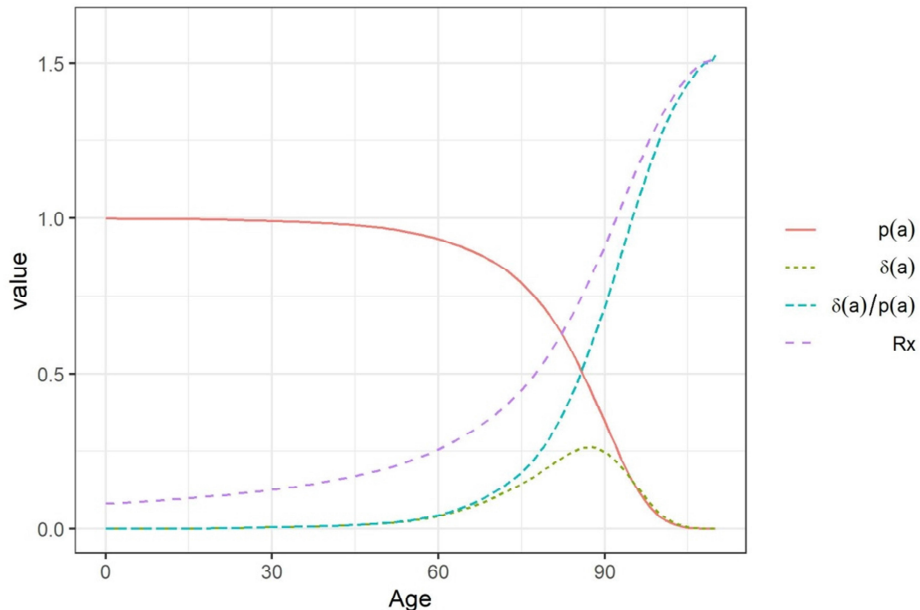
where $R_x = \frac{\int_x^\omega \delta(a) da}{\int_x^\omega p(a) da}$ is the ratio of the cumulative sex difference in survival above age x to the cumulative survival above age x for the total population. When $x = 0$, Equation (9) can be simplified and expressed as Equation (8). Our results indicate that R_x is a strictly increasing function of age x (see Appendix 1 for the detailed proof):

$$\frac{dR_x}{dx} > 0. \quad (10)$$

Figure 1 depicts the age trajectories of $p(a)$, $\delta(a)$, $\frac{\delta(a)}{p(a)}$, and R_x . Prior to reaching middle age (around 50 years), $\delta(a)$ is relatively small (indicated by a solid red line), while $p(a)$ remains at a high level (green dotted line), so their ratio (blue dashed line) is small. After this age, $\delta(a)$ increases with age and peaks at an approximate age of 75 years, with a subsequent gradual decline. At the same time, $p(a)$ declines at a fast pace

at older ages, which are associated with a high mortality rate. Consequently, the value of $\delta(a)/p(a)$ rises rapidly after middle age, implying that the impact of a rise in π_m on population size increases rapidly with advancing age. R_x reveals an upward trend similar to that of $\delta(a)/p(a)$ but at a higher level, with both curves converging at advanced ages.

Figure 1: Age trajectories of functions $p(a)$, $\delta(a)$, $\frac{\delta(a)}{p(a)}$, and R_x , Japan 2010



Source: Human Mortality Database (2020).

We assumed that a constant number of births implies that the fertility rates must rise to some degree to offset the reduction in births resulting from the rise in the SRB. As noted above, a higher proportion of male births implies fewer female births. Consequently, there will be fewer women of childbearing age in the following decades. If the fertility schedules remain unchanged, the decreased number of these women will lead to a further reduction in the number of births, nearly half of which will be female births. This trend will continue across generations, leading to a decreasing number of annual births, which conflicts with the assumption of a fixed number of births. Therefore, fertility must rise to ensure a constant number of annual births.

4. SRB-induced population aging

4.1 A case of fixed annual births

We first prove that a stable female-dominant population with a higher SRB may be younger than one with a lower SRB. Next, we visualize how a rise in the SRB can cause a population to age.

A widely used indicator of population aging is the proportion of the population above the age of 65 years:

$$C_{65}(t) = \frac{\int_{65}^{\omega} N(a,t) da}{\int_0^{\omega} N(a,t) da} = \frac{N_{65+}(t)}{N(t)}. \quad (11)$$

The derivative of C_{65} with respect to π is:

$$\begin{aligned} \frac{d \ln C_{65}(t)}{d \pi_m} &= \frac{d \ln N_{65+}(t)}{d \pi_m} - \frac{d \ln N(t)}{d \pi_m} \\ &= \frac{1}{N_{65+}(t)} \int_{65}^{\omega} \frac{dN(a,t)}{d \pi_m} da - \frac{1}{N(t)} \int_0^{\omega} \frac{dN(a,t)}{d \pi_m} da \\ &= -\frac{\int_{65}^{\omega} \delta(a) da}{\int_{65}^{\omega} p(a) da} - \left(-\frac{\int_0^{\omega} \delta(a) da}{\int_0^{\omega} p(a) da} \right) \\ &= -R_{65} + R_0. \end{aligned} \quad (12)$$

Because R_x is an increasing function of age x , $R_{65} > R_0$ and the right-hand side of Equation (12) has a negative value. In other words, a rise in π_m can lead to a younger population. The reason for this situation is that with increased male births attributed to a rise in the SRB, more individuals in this male cohort will die before reaching old age, leading to a relatively small older population. The total population size will also decrease but at a relatively slow rate, so the proportion of the older population will decrease.

The same logic can be applied to the old-age dependence ratio (OADR), which is another widely used indicator of population aging, as follows:

$$OADR(t) = \frac{\int_{65}^{\omega} N(a,t) da}{\int_{20}^{64} N(a,t) da} = \frac{N_{65+}(t)}{45N_{20}(t)},$$

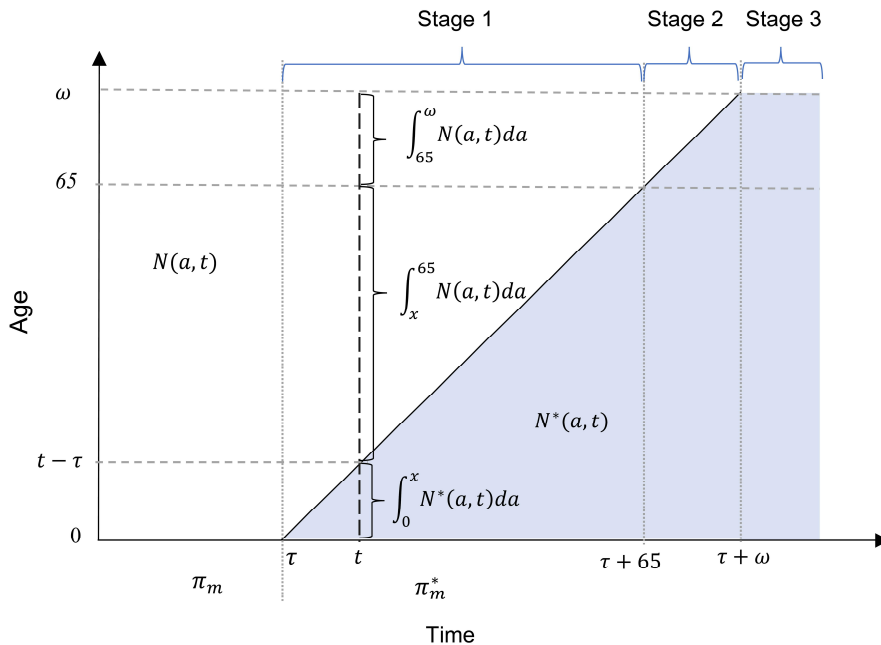
where the denominator is the population's labor force. Similar to Equation (12), we can have

$$\frac{d \ln OADR(t)}{d\pi_m} = -\frac{\int_{65}^{\omega} \delta(a) da}{\int_{65}^{\omega} p(a) da} - \left(-\frac{\int_{20}^{64} \delta(a) da}{\int_{20}^{64} p(a) da} \right) = -R_{65} + {}_{45}R_{20}.$$

It follows from Equation (10) that ${}_{45}R_{20} < R_{20} < R_{65}$ and then the right-hand side of the above equation is negative, which suggests that a rise in the SRB will lead to a decrease in the OADR.

The question that arises is that if a population with a higher SRB is younger than one with a lower SRB, how can a rise in the SRB prompt population aging? The answer to this question can be found in continual changes in the composition of a population in terms of cohorts born during periods with different sex ratios at birth during the transition from a lower SRB period to a higher SRB period (Figure 2).

Figure 2: Graph illustrating how a rise in the SRB alters the composition of a population, and induces its aging



Note: The unshaded area to the left of the diagonal line depicts cohorts born during a period of normal π_m before time τ , denoted as $N(a, t)$. The shaded area to the right of the diagonal line depicts cohorts born during a period of abnormally high π_m after time τ , denoted as $N^*(a, t)$.

If the proportion of male births rises from the normal π_m level to π_m^* , which is an abnormally high level, at time τ , then, as illustrated in Figure 2, the resultant change in the population age structure will occur over three stages until the last individual born with π dies. In the first stage covering the period τ to $\tau + 65$, the population above the age of 65 years, which was born at least 65 years earlier, is not affected by π_m^* , so its size remains unchanged. However, the cohorts born during the π_m^* period are smaller in size than those born before time τ , resulting in a decline in the total population size. Accordingly, the proportion of the elderly will rise. At this stage, the percentage of people aged 65 years and over can be calculated as follows:

$$C_{65}^{(1)}(t) = \frac{\int_{65}^{\omega} N(a,t)da}{\int_0^{t-\tau} N^*(a,t)da + \int_{t-\tau}^{\omega} N(a,t)da}, 0 \leq t - \tau \leq 65, \quad (13)$$

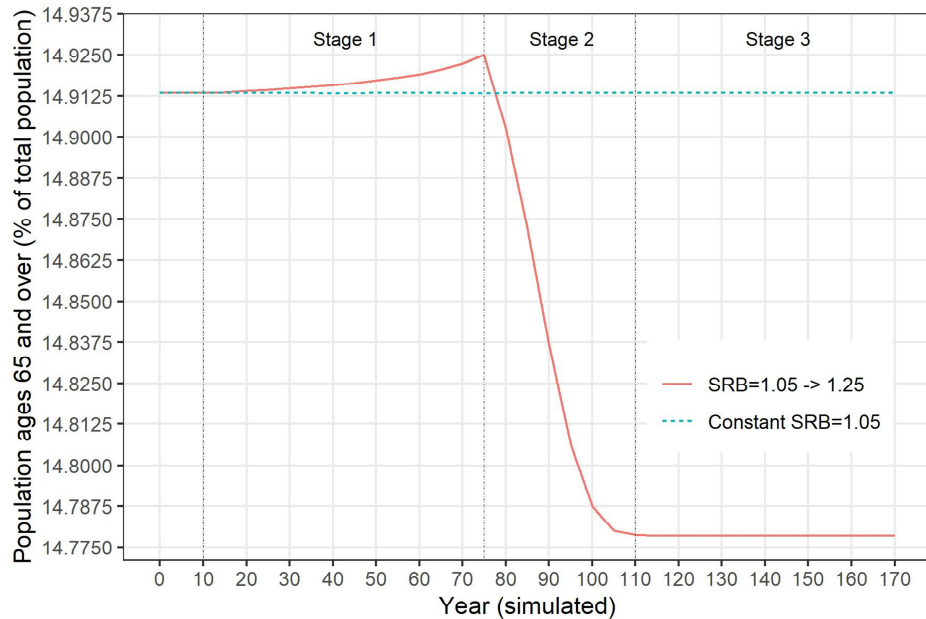
where $N^*(a, t)$ denotes the cohorts born during the new π_m^* period and $N(a, t)$ denotes cohorts born during the normal SRB period. Because $\int_0^{t-\tau} N^*(a, t)da$ is smaller in value than $\int_0^{t-\tau} N(a, t)da$, $C_{65}^{(1)}$ is higher than C_{65} in Equation (11). As more cohorts are born in the π_m^* period, $\int_0^{t-\tau} N^*(a, t)da$ will account for an increasing share of the population, so the denominator of the right hand side of Equation (13) will decrease. Consequently, $C_{65}^{(1)}$ will continue to rise and peak at time $\tau + 65$ when all individuals below 65 years are born during the π_m^* period.

In the second stage that occurs between time $\tau + 65$ and $\tau + \omega$, the proportion of the population above the age of 65 years is calculated as follows:

$$C_{65}^{(2)}(t) = \frac{\int_{65}^{t-\tau} N^*(a,t)da + \int_{t-\tau}^{\omega} N(a,t)da}{\int_0^{t-\tau} N^*(a,t)da + \int_{t-\tau}^{\omega} N(a,t)da}, 65 \leq t - \tau \leq \omega. \quad (14)$$

As cohorts born during the π_m^* period enter old age, the number of older people decreases by $\int_{65}^{t-\tau} N(a, t)da - \int_{65}^{t-\tau} N^*(a, t)da$, where $\int_{65}^{t-\tau} N(a, t)da$ would be the number of older people if they were born during the normal π_m period. This decrease applies to both the numerator and denominator shown in Equation (14), but it can induce a bigger change in the numerator relative to the denominator. Hence, $C_{65}^{(2)}(t) < C_{65}^{(1)}(t)$, which conveys that the population is younger in the second stage than it was in the first stage. As in the case of the expansion, the cohorts born during the π_m^* period, $\int_{t-\tau}^{\omega} N(a, t)da$, will approach zero and $C_{65}^{(2)}(t)$ will evidence a continuous decline.

In the last stage, at time $\tau + \omega$, the population will consist entirely of the cohorts born during the π_m^* period, and the percentage of older individuals within the population will return to a constant level, as defined in Equation (12).

Figure 3: Simulated evolution of population aging with SRB perturbation

Note: Between year 0 and year 10 in the simulation, the SRB remained at 1.05 and then rose to 1.25. The number of annual births was 10,000, and the mortality rates for both sexes were assumed to be constant over time, with a difference of about four years in their life expectancy at birth.

We performed a female-dominant simulation to illustrate SRB-induced population aging. The number of births was estimated by applying the fertility rate only to women (Preston, Heuveline, and Guillot 2001: 121–124). The data on mortality and fertility were derived from the World Population Prospects estimates for China in 1990 (United Nations 2019). Tables with detailed data are provided in Appendices 2 and 3.

For this simulation, we assumed that the normal SRB of 1.05 continued up to year 10, followed by a rise to 1.25. As shown in Figure 3, the percentage of the elderly aged 65 years and over (depicted as a solid red line) increased, peaking at the end of stage 1. Cohorts born during stage 2, entailing an SRB of 1.25, would account for an increasing share of the population, with a corresponding decrease in the size of the elderly population. As previously noted, a reduction in the population size resulting from a rise in the SRB would be much more significant for older cohorts, so their proportions would rapidly decline. At the beginning of stage 3 all members of the population would be born during the period characterized by the new SRB, so the population would reach

a new stationary state with a lower proportion of older individuals than the stationary population born during the period entailing a normal SRB of 1.05.

Despite a clear trend of population aging, the impact of the SRB on the population structure was not significant, with just a small range of variation in percentages of the population above the age of 65 years (14.78%–14.93%). This is mainly because to ensure a fixed number of births the fertility level would need to increase accordingly, and fertility increase will make a population younger, thereby in part offsetting the SRB-induced population aging. In this case, the total fertility rate (TFR) would have to rise from 2.18 to 2.40 to maintain a constant number of annual births.

Although fertility may offset SRB-induced population aging, it should be noted that SRB and fertility have differential effects on the population structure. The SRB can directly impact on the number of females, who represent the future fecundity of a population, whereas fertility rates determine the total number of babies born, nearly half of whom are female, thus impacting indirectly on fecundity. When the total number of births is fixed, the SRB and fertility can become intermeshed with each other.

The different pathways of the impacts of the SRB and fertility imply that they have distinct influences on the growth rate, as shown in the following relationship (Coale 1972:18–22; Dublin and Lotka 1925; Preston et al. 2001:152–154):

$$r = \frac{\ln TFR + \ln \pi_f + \ln p_f(A_M)}{T}, \quad (15)$$

where π_f denotes the proportion of female births, TFR is the total fertility rate, $p_f(A_M)$ denotes the probability of women surviving up to the mean childbearing age (A_M), and T denotes the mean length of a generation. The intrinsic growth rate is an additive function of the log of TFR , π_f , or $p_f(A_M)$. Hence, the effect on the growth rate depends only on the proportionate change in any of the three variables. Hence, the change in r caused by π_f is obtained using the following equations:

$$\Delta r = \frac{\ln(\pi_f^*/\pi_f)}{T} \quad (16a)$$

$$\Delta r = \frac{\ln(TFR^*/TFR)}{T} \text{ or } \Delta r = \frac{\ln(p_f^*(A_M)/p_f(A_M))}{T}, \quad (16b)$$

where the superscript * indicates the new level of the TFR and of $p_f(A_M)$.

Furthermore, it follows from Equation (16) that the three variables can separately yield the same change in growth, although they entail distinct operations relating to population dynamics. Accordingly, specific changes in the three variables are equivalent in terms of their impacts on the growth rate. For instance, if the SRB rises

from 1.05 to 1.25, or, equivalently, if π_f falls by 9.02% from 0.488 to 0.444, and if $T = 27$, then the growth rate will decline by 0.35%. The same change in the growth rate can result from the TFR being reduced by the same percentage (9.02%) from 2.10, which is the fertility replacement level, to 1.91. Analogously, $p(A_M)$ can drop from 0.9844 to 0.8956.

It is noteworthy that an underlying assumption in all of the relations derived using the above equations is that women have survival advantages over men, that is, $p_f(a) > p_m(a)$. In some contexts, however, this assumption may not be relevant because of discriminatory behavior against girls (Guilmoto 2012a). Consequently, female mortality rates may be higher than expected and may even exceed equivalent male rates. When $p_f(a) < p_m(a)$, the formula and the results are interpreted in the reverse manner. China and India, which are two of the ten countries in the world where the mortality rates of girls are higher than expected (Alkema et al. 2014), also feature in the list of 12 countries with significant rises in the SRB (Chao et al. 2019). In China, excess female mortality has decreased since the 1990s (Alkema et al. 2014), making the formula $p_f(a) > p_m(a)$ applicable (United Nations 2019). In India, the mortality rate of girls has long exceeded that of boys. This excess female mortality has even worsened in recent decades (Alkema et al. 2014), and this situation will likely continue throughout this century (United Nations 2019). Therefore, in the case of India, Equation (4) will be positive for young cohorts, with $p_f(a) < p_m(a)$, indicating that a rise in the SRB will increase the number of male births, resulting in the population becoming even younger.

4.2 A case of a fixed fertility regime

An assumption of fixed births facilitates our demonstration of how a rise in the SRB can change the age structure of a population. This assumption, however, is an oversimplification. Hence, we relaxed the assumption regarding the fixed number of births by assuming a time-invariant fertility schedule.

The number of births was obtained by applying women's fertility rates as follows:

$$B(t) = N(0, t) = \int_{\alpha}^{\beta} N_f(a, t) \cdot m(a) da = \int_{\alpha}^{\beta} N_f(0, t) \cdot e^{-ra} \cdot p_f(a) \cdot m(a) da \quad (17)$$

The intrinsic growth rate r is defined by the female fertility and mortality schedules:

$$1 = \int_{\alpha}^{\beta} e^{-ra} \cdot p_f(a) \cdot m_f(a) da, \quad (18)$$

where $p_f(a)$ and $m_f(a)$ apply only to females. The same growth rate can also apply to the male subpopulation because the births are determined by female fertility and mortality within a female-dominant population (Coale 1972: 54). Substituting $N_f(0, t) = N(0, t) \cdot \pi_f$ into Equation (17) yields

$$N(0, t) = \int_{\alpha}^{\beta} N(0, t) \cdot \pi_f \cdot p_f(a) \cdot e^{-ra} \cdot m(a) da.$$

Canceling the $N(0, t)$ from both sides yields

$$1 = \pi_f \cdot \int_{\alpha}^{\beta} e^{-ra} \cdot p_f(a) \cdot m(a) da. \tag{19}$$

Perturbation analysis can be conducted to determine the impact of the SRB on population growth. The sensitivity of r to π_f is obtained as the derivative of r with respect to π_f . If the SRB or the proportion of female births is constant in relation to the age of women, that is, $m_f(a) = \pi_f \cdot m(a)$, then Equation (18) for a one-sex (female) population can be extended to Equation (19) for a two-sex population. The intrinsic growth rate of the male population is identical to that of the female population.

We designated the integral on the right-hand side of Equation (19) as the function $\psi(r)$. We subsequently obtained the following equation according to the rules of implicit function and chain differentiation:

$$\frac{d\psi}{d\pi_f} = \frac{\partial\psi}{\partial r} \frac{\partial r}{\partial\pi_f} + \frac{\partial\psi}{\partial\pi_f}. \tag{20}$$

Because ψ is always equal to 1 and its value does not change with π_f , the left-hand side of Equation (20) should amount to zero. Rearranging the right-hand side of Equation (20), we obtained the following equation:

$$\frac{dr}{d\pi_f} = -\frac{\frac{\partial\psi}{\partial\pi_f}}{\frac{\partial\psi}{\partial r}} = \frac{\int_{\alpha}^{\beta} e^{-ra} \cdot p_f(a) \cdot m(a) da}{\int_{\alpha}^{\beta} \pi_f \cdot a \cdot e^{-ra} \cdot p_f(a) \cdot m(a) da} = \frac{1}{\pi_f A_B} > 0, \tag{21}$$

where $A_B = \int_{\alpha}^{\beta} a \cdot e^{-ra} \cdot p_f(a) \cdot m(a) da$ is the mean age of childbearing in the stable population. Equation (21), in which $dr/d\pi_f > 0$, confirms the above finding that an increase in the proportion of female births can result in increased population growth and vice versa. In a female-dominant population, a rise in the number of girls leads to an increase in fecundity, which will boost population growth even if the TFR remains constant.

Figure 4 illustrates the evolution of population aging when there is a perturbation of the SRB. For the baseline setting, fertility rates were adjusted to 10,000 annual births based on the pattern of age-associated fertility rates in China in 1990. This adjustment yielded results that could be compared to the above-described case entailing a fixed number of births. The mortality was the same as in the above simulation. The SRB remained at the normal level of 1.05 during the first ten years in the simulation, followed by a rise to 1.25. As expected, the proportion of the older population (depicted by a solid red line) rose rapidly and peaked at approximately 16.8%, followed by a decreasing frequency of oscillations before a new level was reached. The oscillations resulted from the perturbation in the numbers of male and female births attributed to the rise in the SRB (Figure 5) (see Coale 1972: 63–65). During the process of convergence towards a stable form the gradually decreasing oscillations in the percentage of older people remained visible until around year 200, indicating that a rise in the SRB has a long-run impact.

Moreover, as shown in Equation (16), specific changes in fertility and mortality can induce the same changes in the population growth rate as are caused by a rise in the SRB. Accordingly, we assessed the relative influence of each of these three factors in altering the age structure of the population. As noted above, for the growth rate to decrease by 9.02% the TFR should reduce from 2.10 to 1.91 and in $p(A_m)$ from 0.9844 to 0.8956. Further, although the SRB is treated as being exogenous to fertility within stable population models, a rise in the SRB can be attributed to a decline in fertility. Therefore, we established a scenario in which a rise in the SRB occurred conjointly with a decline in the TFR.

Of the three factors, a decline in the TFR yielded the highest C_{65} value, which was about two percentage points higher than the baseline level, followed by the rise in SRB, with a reduction in $p(A_m)$ having the least effect. Moreover, the time taken to attain a new equilibrium after a perturbation occurred differed for the three factors: 35 years for $p(A_m)$, 70 years for TFR, and nearly 100 years for the SRB.

The combination of the rise in the SRB and a decline in fertility was evidently considerably more significant than their separate influences. A rise in the SRB increased the level of C_{65} by 1.85 percentage points, while a decline in the TRF raised its level by 2.03 percentage points relative to the baseline. The sum of the two effects was 3.88, whereas the combined effect of the rise in the SRB and a decline in the TFR resulted in an increase of 4.14 percentage points for C_{65} , indicating that the two factors can augment the demographic dynamics. The same combined effect occurred in the case of fixed births. Under the assumption of fixed births, a rise in fertility is needed to ensure a constant number of annual births, which partly offsets the effect of a rise in the SRB. For fixed births and fixed fertility there was a difference of about two percentage

points in the C_{65} , which was indicative of the effect of a rise in the SRB combined with an increase in fertility.

Figure 4: Trajectories of the percentages of the elderly aged 65 years and over

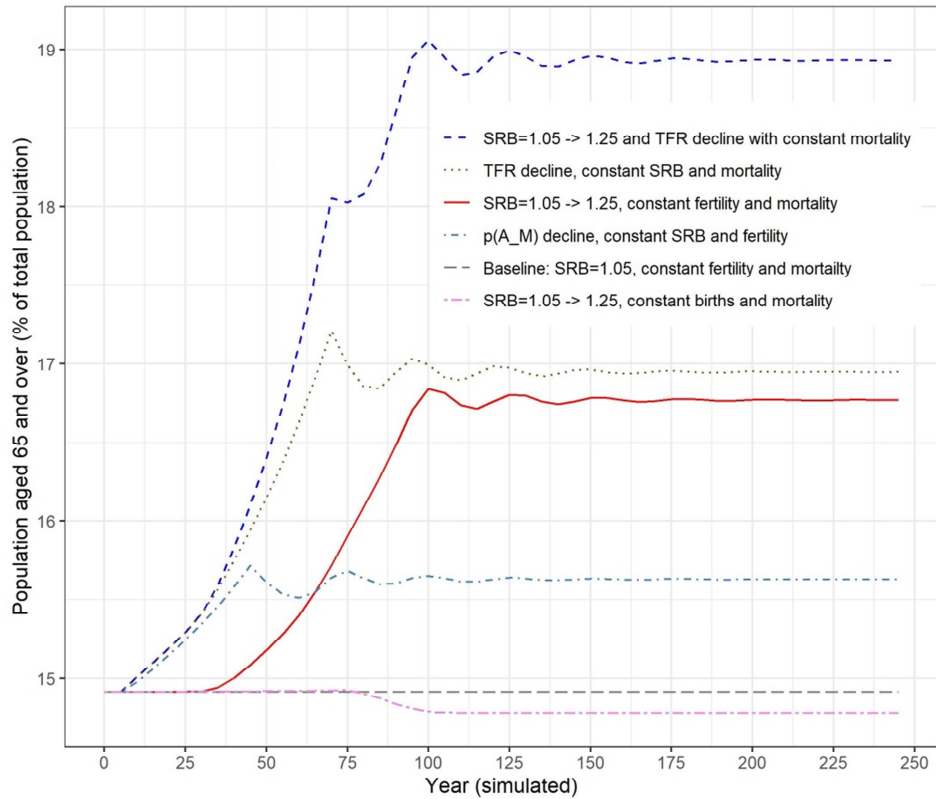
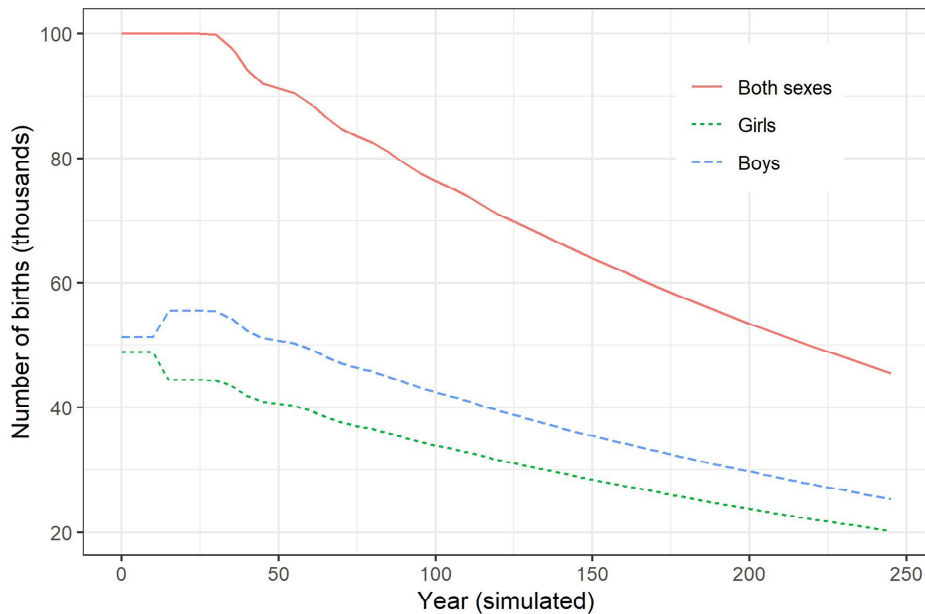


Figure 5: Simulated numbers of births of females, males, and both sexes combined



5. Conclusions

We investigated the role of the SRB in population dynamics, focusing on changes in the age distribution. Using a simple methodological framework derived from classical stable population models, we demonstrated that a change in the SRB can affect the size, structure, and growth of a population. Typically, the rise in the SRB can age a population, and vice versa.

The underlying mechanism is intuitively straightforward. A rise in the SRB will lead to an addition of male infants within the population, who are subject to a higher mortality rate. Thus, more deaths of the new, young cohorts are anticipated. Moreover, a higher proportion of male births implies fewer female births, and therefore fewer women who will give birth in the future. Because the decrease in the number of childbearing women will further reduce the number of births in the next generation, such a process will continue to occur across generations until the population eventually shows a trend towards stability after more than a century. If the number of births is

assumed to be fixed, fertility needs to increase to some degree to ensure a constant number of births. If fertility is assumed to be unchanging the number of births will decline, which will lead to smaller young cohorts. In either case, during the transient period extending from a stationary or stable population with a normal SRB to a period in which the SRB is abnormally high, the population will consist of two groups: older cohorts born during a period with a normal SRB and younger cohorts born during a the period with the new SRB. Because the young cohorts are smaller than they would be if they had been born during the normal SRB period, the population will be older than it was before the increase in the SRB. After a new equilibrium is reached, the population age structure may be younger or older than it was before the SRB rose, depending on which assumption is held: a fixed number of births (implicitly changing fertility) or a fixed fertility regime.

Our findings support those of previous studies, namely that a decline in fertility is the key factor accounting for an aging population and is much more important than a reduction in mortality (Lee and Zhou 2017). Moreover, we found that the SRB is also an important factor accounting for population aging. The effect of a rise in the SRB on C_{65} was slightly smaller than that of a decline in the TFR but much higher than that of a decline in $p(A_m)$. Moreover, our simulations revealed that whereas the impacts of each individual factor on the population age structure was modest, their combined effect, entailing mutual reinforcement within a feedback loop, could have significant demographic consequences for population dynamics.

It should be noted that this study had several limitations. First, whereas the SRB is treated as exogenous to fertility and mortality within stable population models, the rise in the SRB within actual populations is associated with declining mortality and fertility (e.g., Guilmoto 2009 and Kashyap 2019). Further, while we assumed that vital rates are time-constant, in reality they are time-varying. Therefore, there is a dynamic interplay between the resulting population momentum and the changing age structure. Unless the correlation or interaction between these factors is considered, the long-run impact of the distortion of the SRB on population dynamics may be underestimated. Consequently, in a context in which all societies are grappling with the challenges of population aging, much of the focus has been on how to tackle declining fertility and increasing longevity, thereby discounting the SRB.

Second, our analysis was based on a two-sex model in which female dominance was assumed. This assumption holds in most countries where SRB distortions are evident, but it may not apply in extreme situations, such as wars. When there is excess male mortality, a high proportion of women, particularly those of marriageable and childbearing ages, results in a ‘marriage squeeze’ for women. In this case, the rise in the SRB may boost population growth and rejuvenate the population age structure. It

remains an open question as to how marriage can be included in an analysis, enabling deficits relating to one sex, whether male or female, to be handled flexibly.

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References

- Alkema, L., Chao, F., You, D., Pedersen, J., and Sawyer, C.C. (2014). National, regional, and global sex ratios of infant, child, and under-5 mortality and identification of countries with outlying ratios: A systematic assessment. *The Lancet Global Health* 2(9): e521–e530. doi:10.1016/S2214-109X(14)70280-3.
- Babiarz, K.S., Ma, P., Song, S., and Miller, G. (2019). Population sex imbalance in China before the one-child policy. *Demographic Research* 40(13): 319–358. doi:10.4054/DemRes.2019.40.13.
- Cai, Y. and Lavelly, W. (2003). China's missing girls: Numerical estimates and effects on population growth. *China Review* 3(2): 13–29.
- Chahnazarian, A. (1988). Determinants of the sex ratio at birth: Review of recent literature. *Social Biology* 35(3–4): 214–235. doi:10.1080/19485565.1988.9988703.
- Chao, F., Gerland, P., Cook, A.R., and Alkema, L. (2019). Systematic assessment of the sex ratio at birth for all countries and estimation of national imbalances and regional reference levels. *Proceedings of the National Academy of Sciences* 116(19): 9303–9311. doi:10.1073/pnas.1812593116.
- Chen, R. and Zhang, L. (2019). Imbalance in China's sex ratio at birth: A review. *Journal of Economic Surveys* 33(3): 1050–1069. doi:10.1111/joes.12309.
- Chen, W. and Li, M. (2010). Long-term demographic consequence of China's high sex ratio at birth. *Population and Development* (in Chinese) 16(4): 33–37.
- Coale, A.J. (1972). *Growth and structure of human populations: A mathematical investigation*. Princeton, NJ: Princeton University Press: 18–22, 54, 63–65. doi:10.1515/9781400867776.
- Den Boer, A. and Hudson, V. (2017). Patrilineality, son preference, and sex selection in South Korea and Vietnam. *Population and Development Review* 43(1): 119–147. doi:10.1111/padr.12041.
- Dublin, L.I. and Lotka, A.J. (1925). On the true rate of natural increase: As exemplified by the population of the United States, 1920. *Journal of the American Statistical Association* 20(151): 305–339. doi:10.1080/01621459.1925.10503498.
- Guilmoto, C.Z. (2009). The sex ratio transition in Asia. *Population and Development Review* 35(3): 519–549. doi:10.1111/j.1728-4457.2009.00295.x.

- Guilmoto, C.Z. (2012a). Sex imbalances at birth: Current trends, consequences and policy implications. Bangkok: UNFPA-APRO. <http://www.documentation.ird.fr/hor/fdi:010060220>.
- Guilmoto, C.Z. (2012b). Skewed sex ratios at birth and future marriage squeeze in China and India, 2005–2100. *Demography* 49(1): 77–100. doi:10.1007/s13524-011-0083-7.
- Guilmoto, C.Z. (2012c). Son preference, sex selection, and kinship in Vietnam. *Population and Development Review* 38(1): 31–54. doi:10.1111/j.1728-4457.2012.00471.x.
- Guilmoto, C.Z. (2015). Missing girls: A globalizing issue. In: Wright, J.D. (ed.). *International Encyclopedia of the Social and Behavioral Sciences*. London: Elsevier: 608–613. doi:10.1016/B978-0-08-097086-8.64065-5.
- Guilmoto, C.Z. and Ren, Q. (2011). Socio-economic differentials in birth masculinity in China. *Development and Change* 42(5): 1269–1296. doi:10.1111/j.1467-7660.2011.01733.x.
- Human Mortality Database (2020). University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or www.humanmortality.de (data downloaded on 1 July 2020).
- Jiang, Q., Feldman, M.W., and Li, S. (2014). Marriage squeeze, never-married proportion, and mean age at first marriage in China. *Population Research and Policy Review* 33(2): 189–204. doi:10.1007/s11113-013-9283-8.
- Kashyap, R. (2019). Is prenatal sex selection associated with lower female child mortality? *Population Studies* 73(1): 57–78. doi:10.1080/00324728.2018.1442583.
- Kashyap, R. and Villavicencio, F. (2016). The dynamics of son preference, technology diffusion, and fertility decline underlying distorted sex ratios at birth: A simulation approach. *Demography* 53(5): 1261–1281. doi:10.1007/s13524-016-0500-z.
- Lee, R. and Zhou, Y. (2017). Does fertility or mortality drive contemporary population aging? The revisionist view revisited. *Population and Development Review* 43(2): 285–301. doi:10.1111/padr.12062.

- Li, S. (2007). *Imbalanced sex ratio at birth and comprehensive intervention in China*. Paper presented at the 4th Asia Pacific Conference on Reproductive and Sex Health and Rights, Hyderabad, India, October 29–31, 2007.
- Lotka, A.J. (1939). *Théorie analytique des associations biologiques, Part II*. Paris: Hermann et Cie.
- Meslé, F., Vallin, J., and Badurashvili, I. (2007). A sharp increase in sex ratio at birth in the Caucasus. Why? How? In: Attané, I. and Guilmoto, C.Z. (eds.). *Watering the neighbour's garden: The growing demographic female deficit in Asia*. Paris: CICRED: 73–88.
- Preston, S., Heuveline, P., and Guillot, M. (2001). *Demography: Measuring and modeling population processes*. Malden, MA: Blackwell Publishers: 121–124, 152–154.
- Sen, A. (1990). More than 100 million women are missing. *The New York Review of Books* 37(20): 61–66.
- Tuljapurkar, S., Li, N., and Feldman, M.W. (1995). High sex ratios in China's future. *Science* 267(5199): 874–876. doi:10.1126/science.7846529.
- United Nations (2019). *World population prospects: The 2019 revision*. New York: United Nations.

Appendix

Proof for Equation (10).

If $R_x = \frac{\int_x^\omega \delta(a) da}{\int_x^\omega p(a) da}$, then $\frac{dR_x}{dx} > 0$.

Proof:

R_x is differentiated with respect to x as follows:

$$\frac{dR_x}{dx} = \frac{1}{\left(\int_x^\omega p(a) da\right)^2} \left(-\delta(x) \int_x^\omega p(a) da + p(x) \int_x^\omega \delta(a) da \right).$$

The right-hand side of the equation is modified through the substitution of

$$\delta(x) = p_f(x) - p_m(x) \text{ and } p(x) = \pi_m p_m(x) + (1 - \pi_m) p_f(x).$$

Thus, we have:

$$\frac{dR_x}{dx} = \frac{1}{\left(\int_x^\omega p(a) da\right)^2} p_f(x) p_m(x) \left(e_f^o(x) - e_m^o(x) \right).$$

Because $e_f^o(x) > e_m^o(x)$, $\frac{dR_x}{dx} > 0$, which means that R_x is a strictly increasing function of age x .

Table A-1: Life tables for males and females

Male						Female					
x	qx	lx	nLx	Tx	ex	x	qx	lx	nLx	Tx	ex
0	0.0442	1.0000	0.9624	67.43	67.43	0	0.0381	1.0000	0.9671	71.43	71.43
1	0.0107	0.9558	3.7977	66.47	69.54	1	0.0095	0.9619	3.8246	70.46	73.25
5	0.0050	0.9456	4.7161	62.67	66.28	5	0.0035	0.9527	4.7553	66.64	69.94
10	0.0030	0.9409	4.6972	57.96	61.60	10	0.0020	0.9494	4.7422	61.88	65.18
15	0.0040	0.9380	4.6808	53.26	56.78	15	0.0035	0.9475	4.7292	57.14	60.31
20	0.0050	0.9343	4.6598	48.58	52.00	20	0.0040	0.9442	4.7115	52.41	55.51
25	0.0065	0.9296	4.6331	43.92	47.24	25	0.0050	0.9404	4.6903	47.70	50.72
30	0.0075	0.9236	4.6008	39.29	42.54	30	0.0060	0.9357	4.6646	43.01	45.96
35	0.0100	0.9167	4.5607	34.69	37.84	35	0.0075	0.9301	4.6332	38.34	41.22
40	0.0144	0.9076	4.5053	30.13	33.19	40	0.0100	0.9232	4.5929	33.71	36.52
45	0.0198	0.8945	4.4283	25.62	28.64	45	0.0154	0.9140	4.5348	29.12	31.86
50	0.0363	0.8768	4.3044	21.19	24.17	50	0.0242	0.8999	4.4452	24.58	27.32
55	0.0583	0.8450	4.1017	16.89	19.99	55	0.0383	0.8781	4.3068	20.14	22.93
60	0.1002	0.7957	3.7793	12.79	16.07	60	0.0695	0.8446	4.0760	15.83	18.75
65	0.1651	0.7160	3.2844	9.01	12.58	65	0.1114	0.7859	3.7104	11.76	14.96
70	0.2747	0.5978	2.5782	5.72	9.57	70	0.1958	0.6983	3.1497	8.04	11.52
75	0.4013	0.4335	1.7327	3.14	7.25	75	0.2964	0.5616	2.3918	4.90	8.72
80	0.5476	0.2596	0.9425	1.41	5.44	80	0.4612	0.3951	1.5200	2.50	6.34
85	0.7326	0.1174	0.3721	0.47	3.99	85	0.6468	0.2129	0.7202	0.98	4.62
90	0.8969	0.0314	0.0866	0.10	3.08	90	0.8132	0.0752	0.2231	0.26	3.50
95	1.0000	0.0032	0.0101	0.01	3.12	95	1.0000	0.0140	0.0401	0.04	2.86

Table A-2: Age-specific fertility rates

Age	mx
15-19	0.094
20-24	0.945
25-29	0.791
30-34	0.268
35-39	0.057
40-44	0.023
45-49	0.006