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Research Article

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Death distribution methods for estimating adult mortality: Sensitivity analysis with simulated data errors

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Abstract

The General Growth Balance (GGB) and Synthetic Extinct Generations (SEG) methods have been widely used to evaluate the coverage of registered deaths in developing countries. However, relatively little is known about how the methods behave in the presence of different data errors. This paper applies the methods (both singly and in combination) using non-stable populations of known mortality to which various data distortions in a variety of combinations have been applied. Results show that the methods work very well when the only errors in the data are those for which the methods were developed. For other types of error, performance is more variable, but on average, adjusted mortality estimates using the methods are closer to the true values than the unadjusted. The methods do surprisingly well in the presence of typical patterns of age misreporting, though GGB is more sensitive to coverage errors that change with age. The Basic SEG method (that is, making no adjustments for possible change in census coverage) is very sensitive to such coverage change, but the Extended SEG method (that is, adjusting census coverage to obtain a set of completeness estimates that show no trend with age) is little affected. Fitting to the age range 5+ to 65+ is clearly preferable to fitting to 15+ to 55+. Both GGB and SEG are very sensitive to net migration, which is an Achilles heel for all of the methodologies in this paper. In populations not greatly affected by migration, our results suggest that an optimal strategy would be to apply GGB to estimate census coverage change, adjust for it and then apply SEG; in populations affected by migration, applying both GGB and SEG, fitting both to the age range 30+ to 65+, and averaging the results appears best.

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1. Introduction

The study of adult mortality in less developed countries is problematic due to data quality issues. Incomplete vital registration, inaccurate censuses, and misreporting of age at death or age of the living are among the problems often encountered by researchers wishing to use these data-sets (United Nations 1983, 2002; Bhat 1990). Considerable ingenuity has been shown in the development of methods to estimate adult mortality despite these data challenges. There are three broad groups of methods for evaluating data quality or otherwise estimating adult mortality: (1) death distribution methods that assess the completeness of death recording relative to census recording, (2) methods based on intercensal survival, and (3) methods that convert indicators of mortality levels based on survival of close relatives into standard life table functions.

Where the necessary data exist, death distribution methods are the method of choice because they provide age-period specific estimates of mortality rates (Hill 2001). These methods compare the distribution of deaths by age with the age distribution of the living and provide the age pattern of mortality in a defined reference period. Standard methods require two population censuses (or large sample surveys) to provide age distributions of the living and the changes of such distributions over time, plus information to calculate an age pattern of deaths for the intercensal period. If the completeness of death recording relative to population recording can be estimated, and there are no other data errors, any differential in completeness can be adjusted for, and unbiased death rates and standard life table functions calculated. However, the methods require numerous assumptions about the population they are applied to and about the nature of typical data errors. Standard methods assume the population to experience no net migration. Strong simplifying assumptions are made about data errors: no age misreporting (of either population or deaths), proportionately constant omission of deaths by age (an assumption that also implies no selectivity bias in deaths that are reported) and that any change in census coverage has been proportionately constant by age.

Little is known about how the methods are affected by errors that do not match these assumptions. The purpose of this paper is to explore the sensitivity of the methods to deviations from these assumptions through application to data sets that include known errors. The paper builds on earlier research by Hill and Choi (2004) and Dorrington, Timæus, and Moultrie (2008).

2. Death distribution methods

The distribution of deaths by age and the distribution of the population by age are linked via growth rates in various identities that provide a basis for consistency checks. There are two major approaches - (1) the General Growth Balance (GGB) method and (2) the

Synthetic Extinct Generations (SEG) method. These methods are briefly reviewed below.

We do not explicitly examine the performance of a "short cut" method proposed by Preston and Lahiri (1991), though some trial analyses suggest this method will be affected by errors in much the same way as the basic SEG method.

2.1 The General Growth Balance Method

Brass (1975) first proposed the Growth Balance method, deriving from stable population equations the intuitively-necessary relationship that, for any open-ended age segment $a+$ of a closed population, the entry rate into the segment ($b(a+)$) is equal to the growth rate of the segment ($r(a+)$) plus the exit (death) rate ($d(a+)$) of the segment. The corollary for the whole (closed) population is of course that the growth rate is equal to the birth rate minus the death rate. Thus

$$r(a+) = b(a+) - d(a+) \quad \text{or} \quad b(a+) = r(a+) + d(a+) \qquad (1)$$

In a stable population, the growth rate is constant for all segments, so the entry rates and the death rates must be linearly related. If we write $N(a)$ and $N(a+)$ for the number of entries (that is, birthdays at age a) into, and the population of, the age group a and over respectively, r as the stable population growth rate, and $D(a+)$ as the deaths at ages a and over,

$$\frac{N(a)}{N(a+)} = r + \frac{D(a+)}{N(a+)} \qquad (2)$$

If the entry rate is calculated from a population age distribution alone using fairly simple approaches such as obtaining $N(a)$ as one-fifth of the average of the five-year populations under and over age a, any population coverage error that is invariant with age cancels out, whereas the death rate, calculated from both deaths by age and population by age, will be affected by any differential coverage between population and deaths. The slope of the line relating the entry rate to the exit rate will estimate the completeness of population recording relative to death recording and provide a potential adjustment factor for the deaths.

$$\frac{N^o(a)}{N^o(a+)} = r + \frac{1}{c} * \frac{D^o(a+)}{N^o(a+)} \qquad (3)$$

where superscript ^{o} refers to observed values, $N^o(a)/N^o(a+)$ is the entry rate, $D^o(a+)/N^o(a+)$ is the observed death rate, r is the stable population growth rate, and c is the completeness of death recording relative to population recording (assumed constant by age).

This simple method can be generalized for non-stable populations when two or more census enumerations are available (Hill 1987). The growth rate of each segment can then be calculated from the census counts, and the assumption of stability is no longer needed. The relationship of the entry rate minus the growth rate to the death rate estimates (1) an intercept that captures any age-invariant change in census coverage between the two censuses and (2) a slope that estimates the coverage of death recording relative to an average of the coverage of the two censuses.

$$\frac{N^o(a)}{N^o(a+)} - r^o(a+) = k + \frac{1}{c} * \frac{D^o(a+)}{N^o(a+)} \tag{4}$$

where $r^o(a+)$ is the observed growth rate of the population *a* and over, and *k* is the error in the growth rate (assumed constant across ages), arising, for instance, from a systematic change in census coverage between the first and the second census.

The method requires three major assumptions: (1) a closed population, (2) invariant coverage of population and deaths by age within but not across sources, and (3) accurate recording of age for both population and deaths.

2.2 The Synthetic Extinct Generations Method

The Synthetic Extinct Generation method (Bennett and Horiuchi 1981, 1984) is based on the insight of Vincent (1951) that, in a closed population with perfect recording of deaths, the population age *a* at time *t* could be estimated by accumulating the deaths to that cohort after time *t* until the cohort was *extinct*. This is equivalent to the life table relationship that

$$\ell(a) = \sum_{x=a}^{\varpi} d(x) \tag{5}$$

Thus in a stationary population, period deaths above age *a* are equal to the population of exact age *a*. Bennett and Horiuchi generalized the method to non-stable closed populations by using age-specific growth rates. The population at age *a* can be estimated from the period deaths at all ages *x* above that age *a* by applying exponentiated summed age-specific growth rates from *a* to *x* to allow for the demographic history of the population:

$$N(a) = \int_{x=a}^{\varpi} D(x) e^{\int_a^x r(y)dy} dx \qquad (6)$$

The ratio of the population age *a* estimated in this way from the deaths to the observed population age *a* estimates the completeness of death recording (assumed constant at all ages) relative to census coverage:

$$\hat{c}(a) = \frac{\hat{N}(a)}{N^o(a)} = \frac{\int_a^{\varpi} D^o(x) e^{\int_a^x r(y)dy} dx}{N^o(a)} \qquad (7)$$

where $\hat{c}(a)$ is the estimated coverage of deaths above age *a* relative to population and $\hat{N}(a)$ is the estimated population aged *a* derived from deaths and growth rates above age *a*. In its basic form, the SEG method adds an additional assumption -- invariant coverage of population across time -- to the three assumptions required in the General Growth Balance method: (1) a closed population, (2) invariant coverage of population and deaths by age, and (3) accurate recording of age for both population and deaths. However, Bennett and Horiuchi (1981) suggest that the problem of change in census coverage (and thus biased growth rates at all ages) can be addressed by iteratively adjusting one census count or the other by a constant factor until the plot of completeness estimates *c(a)* is as horizontal across some age range as possible; we refer to this as the “extended SEG” method. The problem can also be addressed by combining the SEG method with the GGB: first estimating change in census coverage using GGB, then adjusting the census data for the estimated coverage change, and then applying the SEG method; we refer to this as the “combined GGB-SEG” approach.

3. Implementation

Both GGB and SEG require simplifying assumptions when working with discrete data, broad age groups and non-instantaneous transition rates. GGB needs estimates of $N^o(a)$, $N^o(a+)$, $r^o(a+)$ and $D^o(a+)$, which are estimated from successive census counts *N1* and *N2* separated by *t* years and annual counts of deaths by age as follows:

$$N^o(a) = \left(\frac{1}{5}\right)\sqrt{{}_5N1^o_{a-5} * {}_5N2^o_a} \qquad (8a)$$

$$N^o(a+) = \sum_{a}^{\varpi-5} \sqrt{{}_5N1^o_y * {}_5N2^o_y} \qquad (8b)$$

$$r^o(a+) = \left(\frac{1}{t}\right)\ln\left(\frac{N2^o(a+)}{N1^o(a+)}\right) \quad (8c)$$

$$D^o(a+) = \left(\frac{1}{t}\right)\sum_{x=a}^{\varpi-5} {}_5D^o{}_{x,t} \quad (8d)$$

SEG needs estimates of N^o *(a)* (for which equation 8a is used) and a way of approximating the integral of the growth rate for five year age groups. The simplifying approach used is

$$\int_a^x r(y)dy = 5 * \sum_a^x {}_5r_y + 2.5 * {}_5r_x \quad (8e)$$

For the open interval *a*+, factors proposed by Bennett and Horiuchi (1981) are used.

In practice, points for GGB do not lie on a perfect straight line, and SEG estimates of coverage are not constant across ages, so fitting procedures or averaging is required. Two fitting age ranges were tested: 15+ to 55+ and 5+ to 65+. We avoided using ranges starting or terminating in ages ending in zero because of the expectation that digital preference for such ages would distort results more than for ages ending in five. We chose the age range 15+ to 55+ because it approximates the age range of our main summary index of adult mortality, the probability of dying between the ages of 15 and 60 (${}_{45}q_{15}$), without using the age point 60+ which can be expected to be distorted by major digital preference for age 60; it has also been argued that there is relatively little age displacement across age 15 (United Nations 1983). We chose the age range 5+ to 65+ because it covers late child and most adult experience. We do not use terminal ages above 65 because of the expectation that age misreporting will be a more serious problem at older ages, an expectation confirmed by some trials with the simulated data sets up to 75+.

The intercept and slope of the GGB method were obtained by orthogonal regression to points in the selected age ranges. The coverage estimate *c* of deaths for both the basic and the extended SEG methods was obtained by averaging the estimates *c(a)* for the required age range; for the extended SEG method, a census coverage adjustment was found by iteratively adjusting coverage of the first census (relative to the second) until the regression line for *c(a)* on *a* had a slope of zero for the range of *a* in question. Finally the two step GGB-SEG procedure was applied by using the GGB intercept estimate to adjust the data for the estimated census coverage change and then applying the SEG approach.

4. Simulations

In order to test the effects of deviations from the idealized assumptions imposed by the death distribution methods, we start with an initial non-stable population and a set of known rates of fertility and mortality, and project the population for a simulated intercensal interval of five years. Two population scenarios are built by using different initial populations and rates: population A starts with a young non-stable population and projects forward in terms of a given age-specific fertility schedule (Total Fertility Rate of 5.0) and the level 15 mortality schedule of Coale-Demeny's (1983) West female model life table (*e*(0) 55 years, ${}_{45}q_{15}$ of 0.309, ${}_{25}q_{60}$ of 0.860) to get the population after five years; population B starts with the population of the developing world in 1995 according to the United Nations (2005) *World Population Prospects 2004 Revision*, and is projected for five years using the estimated fertility schedule for developing countries in 1995-2000 and the level 19 mortality schedule of Coale-Demeny's (1983) West female model life table (e(0) of 65 years, ${}_{45}q_{15}$ of 0.204, ${}_{25}q_{60}$ of 0.810).

The performance of the death distribution methods when data are not perfect or their assumptions are not met is tested by building two error sets (error set I and II) with 6 data-error categories into each projected population, with a design that the data errors in error set I are larger than those in error set II (Table 1). In error set I, we set up a 4% increase in census coverage of the population from the first to the second census; deaths were affected by a 30% omission; age misreporting in population and/or in deaths derived from a matrix of transfers between 5-year age groups for Nigeria in 1969 estimated by Caldwell and Igun (1971); age-varying census coverage based on net-undercount of the male African American population in the 1980 U.S. census estimated by Preston et al. (1998); age-varying coverage of death was assumed as a linear increase or decrease over age; and emigration or immigration was based on an age pattern of net migration to Beijing during the 1990s approximating a net migration rate of about 8 per 1,000 population. In error set II, we use a 2% change of census coverage, a 20% death omission, age misreporting in population and/or in deaths derived from an age misreporting matrix for India estimated by Bhat (1990); a linearly increasing or decreasing coverage of deaths with age but with slopes smaller than those in error set I, and an age pattern of migration to the U.S. of Mexicans in 1980-1990 approximating a net migration rate of 2 per 1,000.

Table 1: Error sets

Data Error Category	*Error set I*	*Error set II*
Census coverage change	4%	2%
Death omission	30%	20%
Age misreporting	Age misreporting matrix for India 1971-1981 estimated by Bhat (1990)	Age misreporting matrix for Nigeria in 1969 estimated by Caldwell and Igun (1971)
Age varying coverage of census	Age varying coverage for male African American population in U.S. 1980 census estimated by Preston et al. (1998);average absolute proportionate error 0.076	Age varying coverage in India in 1981 census estimated by Bhat (1990); average absolute proportionate error 0.026
Age varying coverage of death	Linear decreasing or increasing	Linear with slopes lower than those in error set I
Emigration or Immigration	Age pattern of net migration of Beijing during 1990s (net migration rate around 0.8%)	Age pattern of in-migration to U.S. of Mexican 1980-1990 (net migration rate around 0.2%)

Rather than test all possible combinations of error categories, we carried out simulations for each error category individually (with all other data correct) in order to identify the effect of a particular error on its own, and then combined error categories in groups based on expectations of how data errors would occur in practice. The result was a total of 24 selected data error patterns built into each of the two error sets (Table 2). Notice that error types 1, 2 and 3 do not violate the underlying assumptions of the GGB and extended SEG methods, and error type 2 does not violate the assumptions of the basic SEG method. Across both error sets, a total of 96 error simulations were conducted. In addition there is also one test with no error for each projected population (essentially testing the simplifying assumptions required for implementation), for a total of 98 simulations.

For each simulation, both the GGB and the SEG methods were applied. As a summary measure, both unadjusted and adjusted probabilities of dying between 15 and 60 years of age (${}_{45}q_{15}$) were estimated for each case to represent adult mortality, and probabilities of dying between 60 and 85 years of age (${}_{25}q_{60}$) were estimated to measure mortality of the elderly.

Table 2: Error types

Error Type	*Age misreporting*		*Coverage change of census2 compared to census 1*	*Age varying coverage for census*	*Age varying coverage for death*		*Deaths omission*	*Migration*	
	Census	Deaths			Increasing with age	Decreasing with age		Emigration	Immigration
1			X						
2							X		
3			X				X		
4	X								
5		X							
6	X	X							
7				X					
8					X		X		
9						X	X		
10	X			X					
11			X	X					
12	X	X	X						
13	X	X					X		
14	X	X	X				X		
15								X	
16			X					X	
17							X	X	
18	X		X					X	
19	X	X	X				X	X	
20									X
21			X						X
22							X		X
23	X		X						X
24	X	X	X				X		X

5. Results

Results indicate that errors were little affected by the initial population age distribution, and also varied relatively little by error model (though errors were quantitatively larger for error set 1 than for error set 2, they were not qualitatively different). To simplify, we therefore present (for each error type) the average percentage error across population type and error set for each of the four methods and each of the age ranges used for final estimation. Table 3 shows by scenario the errors in ${}_{45}q_{15}$, and Table 4 the errors in ${}_{25}q_{60}$. Three summary measures across scenarios are presented: the median error, the mean error and the root mean square error. The median and mean indicate potential bias, with the mean giving more emphasis to outliers, whereas the root mean square error indicates the magnitude of error regardless of direction.

The good news is that all the methods work well for the data problems they were designed for. For no error and error types 1 to 3 (omission of deaths, changes in census coverage, all proportionately equal) the errors in both ${}_{45}q_{15}$ and ${}_{25}q_{60}$ are all less than ± 1 percent for GGB, Combined GGB-SEG and Extended SEG methods, though the Basic SEG method is affected by large errors when census coverage changes. Further good news is that error types 4 to 6 (age misreporting in population or deaths) also give rise to quite small net errors in ${}_{45}q_{15}$, generally less than ± 2.5%, though the errors in ${}_{25}q_{60}$ are greater for errors 4 and 6. The methods fitted to the age range 5+ to 65+ also cope well with error 7, age-varying census coverage, though fitting to the age range 15+ to 55+ does much less well (because the census coverage model concentrates errors among young adults). The same is true of the composite error types 10 through 14: with the exception of the Basic SEG method, which is thrown off by all the combinations including change in census coverage, all the methods using points for 5+ to 65+ give rise to errors in ${}_{45}q_{15}$ of less than ± 3%, though the errors in ${}_{25}q_{60}$, mostly negative, range up to 8%.

The bad news is that all the methods give rise to double digit percentage errors in ${}_{45}q_{15}$ (but only small errors in ${}_{25}q_{60}$) when completeness of death recording increases or decreases with age (error types 8 and 9), and all except the Basic SEG give rise to double digit percentage errors in ${}_{45}q_{15}$ for all the scenarios including immigration or emigration (error types 15 to 24). Errors average close to ± 20% for the age range 5+ to 65+, and are close to 25% for GGB fitted to the age range 15+ to 55+, though the errors are smaller in absolute terms for ${}_{25}q_{60}$. The Basic SEG method does well in the scenarios combining emigration with increasing census coverage (Error types 16, 18 and 19), because of errors canceling out, and would no doubt also do well in scenarios combining immigration and declining census coverage, though we did not examine such scenarios.

Table 3: Average percentage errors in $_{45}q_{15}$ by method, error type and fitting range

Error Type*	Fitting to Age Range 15+ to 55+				
	Observed	Adjusted			
		GGB	Basic SEG	Combined GGB-SEG	Extended SEG
No Error	0.0	-0.3	0.0	-0.3	-0.2
Error 1	1.3	-0.3	-17.8	-0.3	-0.4
Error 2	-22.3	-0.3	0.1	-0.3	0.1
Error 3	-21.3	-0.3	-17.8	-0.3	-0.2
Error 4	2.6	-0.9	2.1	1.7	2.1
Error 5	0.8	2.3	0.1	0.7	0.3
Error 6	3.0	1.5	2.1	2.5	2.4
Error 7	6.2	-10.8	0.2	-5.6	-5.5
Error 8	-18.1	-14.4	-12.1	-15.2	-15.1
Error 9	-26.7	23.4	19.0	25.0	25.4
Error 10	8.7	-9.3	2.4	-2.7	-2.7
Error 11	7.7	-9.6	-18.5	-4.8	-4.9
Error 12	4.3	1.5	-16.4	2.5	2.3
Error 13	-20.0	1.5	2.1	2.5	2.3
Error 14	-18.9	1.5	-16.4	2.5	2.2
Error 15	0.0	-24.5	12.5	-18.3	-21.9
Error 16	1.3	-24.5	-6.3	-18.3	-21.8
Error 17	-22.3	-24.5	14.0	-18.3	-20.6
Error 18	3.9	-25.3	-4.0	-16.5	-19.2
Error 19	-18.9	-23.7	-4.2	-16.5	-19.8
Error 20	0.0	26.3	-12.3	23.8	24.3
Error 21	1.3	26.3	-28.6	23.8	24.0
Error 22	-22.3	26.3	-12.3	23.8	24.3
Error 23	3.9	25.2	-27.1	25.3	25.9
Error 24	-18.9	28.3	-27.8	27.1	27.3
Median	0.0	-0.3	-4.2	-0.3	0.1
Mean	-6.6	-0.2	-6.7	1.8	1.2
RMSE	0.141	0.200	0.150	0.169	0.181

Table 3: (Continued)

Error Type*	Fitting to Age Range 5+ to 65+				
	Observed	Adjusted			
		GGB	Basic SEG	Combined GGB-SEG	Extended SEG
No Error	0.0	-1.0	0.0	-0.8	-0.7
Error 1	1.3	-1.0	-18.2	-0.8	-0.7
Error 2	-22.3	-1.0	0.0	-0.8	-0.5
Error 3	-21.3	-1.0	-18.2	-0.8	-0.6
Error 4	2.6	-1.7	2.6	0.9	0.6
Error 5	0.8	2.9	0.1	0.9	0.1
Error 6	3.0	1.6	2.6	2.3	1.2
Error 7	6.2	0.4	0.9	1.3	-0.5
Error 8	-18.1	-15.7	-12.2	-16.0	-15.7
Error 9	-26.7	24.4	19.2	25.6	25.4
Error 10	8.7	-0.4	3.6	3.0	1.1
Error 11	7.7	1.4	-18.4	2.1	0.2
Error 12	4.3	1.6	-16.2	2.3	1.1
Error 13	-20.0	1.6	2.6	2.3	1.2
Error 14	-18.9	1.6	-16.2	2.3	1.1
Error 15	0.0	-16.8	14.9	-14.7	-19.4
Error 16	1.3	-16.8	-3.9	-14.7	-19.2
Error 17	-22.3	-16.8	16.4	-14.7	-18.0
Error 18	3.9	-18.1	-1.2	-13.0	-17.3
Error 19	-18.9	-15.5	-1.3	-12.6	-17.7
Error 20	0.0	16.0	-15.2	16.0	18.7
Error 21	1.3	16.0	-31.8	16.0	13.7
Error 22	-22.3	16.0	-15.2	16.0	18.9
Error 23	3.9	15.5	-29.6	17.6	19.3
Error 24	-18.9	19.5	-30.4	20.0	20.9
Median	0.0	0.4	-1.3	1.3	0.2
Mean	-6.6	0.5	-6.6	1.6	0.5
RMSE	0.141	0.135	0.162	0.131	0.149

* See Table 2 for descriptions.
GGB: General Growth Balance; SEG: Synthetic Extinct Generations; RMSE: root mean square error.

Table 4: Average percentage errors in $_{25}q_{60}$ by method, error type and fitting range

Error Type*	Fitting to Age Range 15+ to 55+				
	Observed	Adjusted			
		GGB	Basic SEG	Combined GGB-SEG	Extended SEG
No Error	0.0	-0.1	0.0	-0.1	-0.1
Error 1	0.6	-0.1	-9.2	-0.1	-0.2
Error 2	-11.8	-0.1	0.0	-0.1	0.0
Error 3	-11.2	-0.1	-9.2	-0.1	-0.1
Error 4	-3.7	-5.5	-4.0	-4.2	-4.0
Error 5	-1.4	-0.7	-1.7	-1.5	-1.7
Error 6	-7.1	-8.1	-7.6	-7.4	-7.4
Error 7	0.1	-8.8	-2.5	-5.6	-5.5
Error 8	-2.0	0.0	1.1	-0.4	-0.4
Error 9	-25.9	-0.3	-2.0	0.2	0.4
Error 10	-3.7	-12.6	-6.5	-9.0	-9.2
Error 11	0.6	-8.1	-12.7	-5.2	-5.3
Error 12	-6.5	-8.1	-17.3	-7.4	-7.5
Error 13	-19.5	-8.1	-7.6	-7.4	-7.5
Error 14	-18.8	-8.1	-17.3	-7.4	-7.5
Error 15	0.0	-13.8	4.9	-9.6	-12.0
Error 16	0.6	-13.8	-3.0	-9.6	-12.0
Error 17	-11.8	-13.8	4.8	-9.6	-11.9
Error 18	-3.1	-19.1	-6.8	-13.7	-15.7
Error 19	-18.8	-21.8	-10.6	-17.4	-19.6
Error 20	0.0	8.8	-6.0	8.1	8.2
Error 21	0.6	8.8	-16.3	8.1	8.1
Error 22	-11.8	8.8	-6.0	8.1	8.2
Error 23	-3.2	3.8	-20.7	4.1	4.5
Error 24	-18.8	1.4	-24.7	1.3	1.4
Median	-3.7	-5.5	-6.5	-4.2	-4.0
Mean	-7.1	-4.8	-7.2	-3.4	-3.9
RMSE	0.110	0.112	0.109	0.086	0.094

Table 4: (Continued)

Error Type*	Fitting to Age Range 5+ to 65+				
	Observed	Adjusted			
		GGB	Basic SEG	Combined GGB-SEG	Extended SEG
No Error	0.0	-0.4	0.0	-0.3	-0.3
Error 1	0.6	-0.4	-9.4	-0.3	-0.3
Error 2	-11.8	-0.4	0.0	-0.3	-0.2
Error 3	-11.2	-0.4	-9.4	-0.3	-0.3
Error 4	-3.7	-5.7	-3.7	-4.5	-4.7
Error 5	-1.4	-0.4	-1.7	-1.4	-1.7
Error 6	-7.1	-7.7	-7.3	-7.4	-8.0
Error 7	0.1	-2.5	-2.2	-2.1	-2.9
Error 8	-2.0	-0.7	1.0	-0.8	-0.7
Error 9	-25.9	0.0	-1.9	0.4	0.4
Error 10	-3.7	-7.9	-6.0	-6.3	-7.2
Error 11	0.6	-2.1	-12.6	-1.8	-2.6
Error 12	-6.5	-7.7	-17.2	-7.4	-8.0
Error 13	-19.5	-7.7	-7.3	-7.4	-7.9
Error 14	-18.8	-7.7	-17.2	-7.4	-8.0
Error 15	0.0	-8.7	5.7	-7.4	-10.4
Error 16	0.6	-8.7	-1.8	-7.4	-10.3
Error 17	-11.8	-8.7	5.6	-7.4	-10.3
Error 18	-3.1	-14.9	-5.5	-11.9	-14.6
Error 19	-18.8	-16.9	-9.2	-15.2	-18.3
Error 20	0.0	5.9	-7.7	5.9	6.7
Error 21	0.6	5.9	-18.7	5.9	5.2
Error 22	-11.8	5.9	-7.7	5.9	6.7
Error 23	-3.2	1.3	-22.7	2.1	2.6
Error 24	-18.8	-0.6	-26.7	-0.5	-0.3
Median	-3.7	-2.1	-7.3	-1.8	-2.6
Mean	-7.1	-3.7	-7.3	-3.1	-3.8
RMSE	0.110	0.076	0.115	0.068	0.083

* See Table 2 for descriptions.
GGB: General Growth Balance; SEG: Synthetic Extinct Generations; RMSE: root mean square error.

Median and mean error across all error types are small for ${}_{45}q_{15}$ except for the Basic SEG, but this is small comfort in practice since a particular data set will rarely have just one set of problems; for ${}_{25}q_{60}$ the medians and means are somewhat larger in absolute terms, tend to be negative, and are smaller (in absolute terms) when estimates are based on the age range 5+ to 65+ than 15+ to 55+. Of more interest for trying to choose between methods is the root mean square error (RMSE), calculated on the basis of individual results from both populations and error patterns rather than from the averages in the tables: except for Basic SEG, the RMSEs suggest that the fitting range 5+ to 65+ should be preferred over 15+ to 55+, and that the combined GGB-SEG approach has the lowest error, though only by a small margin.

In summary, the methods work very well when their underlying assumptions are met, and quite well in the presence of typical patterns of age misreporting, either in the censuses or in deaths. None of the methods work well with death coverage that changes systematically with age, and all of them can produce seriously distorted estimates in the presence of migration (though Basic SEG can give reasonable estimates when migration and change in census coverage operate in different directions). Fitting to the age range 5+ to 65+ is preferable to using the age range 15+ to 55+. Using the root mean square error as the criterion of performance, the GGB-SEG approach (for the fitting age range of 5+ to 65+) performs best overall for both ${}_{45}q_{15}$ and ${}_{25}q_{60}$, although median and mean errors are smaller for the Extended SEG and the original GGB methods.

6. Discussion

The death distribution methods (Growth Balance and Synthetic Extinct Generations) developed to estimate the coverage of death recording have been widely used. However, there has been no systematic evaluation of how they respond to different types of error, and no consensus reached on which method should be preferred, what age range should be used for arriving at a final estimate, or how patterns in the diagnostic plots should be interpreted. The purpose of this paper has been to apply the methods to populations with known parameters on which a variety of simulated errors have been imposed, combined in various ways, to provide guidance in answering these questions.

The results are reassuring in that the methods work very well when their assumptions are met, and also turn out to be quite robust to typical patterns of age misreporting. The results are disturbing, however, in that the methods generally work poorly when death coverage varies by age and when the population is affected by migration. If no information is available to guide the analyst, the strategy that appears best (in terms of minimizing RMSE) on the basis of the error patterns explored in this paper is to use the combined GGB-SEG approach, first applying GGB to estimate change in census coverage, adjusting for the estimated change, and then applying SEG, using the age range 5+ to 65+ for fitting purposes.

The conclusion above that the combined GGB-SEG approach is the safest choice in the absence of other information about errors disagrees with the conclusion of Dorrington et al. (2008), who find that a slightly different implementation of the Extended SEG method was closest to correct in 15 out of 23 of their scenarios; whereas the Combined GGB-SEG approach was closest in only five. Though it is not clear whether this difference arises from a difference in scenarios, a difference in or implementation of the Extended SEG approach, or from a different criterion; however, the last seems the most likely explanation. In our simulations, the Extended SEG is closest to the correct answer in 16 of 25 scenarios, whereas the Combined GGB-SEG approach is closest in only nine. The advantage of the Combined GGB-SEG approach in terms of RMSE indicates that its worst results are not quite as bad as those of the Extended SEG.

6.1 Adjusting for migration

Migration rates are typically highest for young adults, and drop sharply in middle age. Both the GGB and SEG methodologies use information on deaths by age above some age (or series of ages). One possible approach to limiting errors arising from net migration is therefore to use a high starting age, say 30 or 35, for the fitting range. Such an approach may reduce the effects of migration, but may also increase the effects of, for example, age misreporting that may get worse with increasing age. It is also interesting to note that the GGB and the Basic SEG methods are biased in different directions by migration, although the migration bias can be outweighed in the Basic SEG method by a quite small countervailing change in census coverage (see error scenarios 16, 18 and 19 in Table 3).

The GGB, combined GGB-SEG and Extended SEG methods underestimate coverage (overestimate adjusted mortality) in populations affected by immigration, whereas the basic SEG method does the reverse.

Fitting to the age range 30+ to 65+ reduces the RMSE across all scenarios for both GGB and Basic SEG by about 40% ; the errors for the migration scenarios are in general reduced by two-thirds or so (results not shown). The general level of error increases slightly with the shorter range, but mostly in the 2% to 3 % range; an exception is for GGB in the presence of age-varying census coverage, for which the narrower age range gives much worse results. Fitting both GGB and Basic SEG to the age range 30+ to 65+ and then averaging the estimates results in the smallest RMSE across all error scenarios; the two largest absolute errors are an overestimate of ${}_{45}q_{15}$ of 27% with omission of deaths but decreasing omission by age, and an underestimate of 16% with omission of deaths and increasing omission with age. The errors in the scenarios affected by migration are typically less than 10%.

7. Conclusion

The application of death distribution methods to populations with simulated errors indicates that the methods are very effective at allowing for the errors for which they were designed, but can be very sensitive to errors for which they were not designed, particularly substantial migration and age-specific changes in propensities to report either deaths or population.

In populations thought not to be affected by migration, the optimal strategy appears to be to apply the General Growth Balance method (fitting to the age range 5+ to 65+) to estimate census coverage change, adjust one or other of the two population age distributions for the estimated coverage change, and then apply the Synthetic Extinct Generations method to the adjusted data, also fitting to the age range 5+ to 65+. In populations thought to be experiencing substantial migration, applying either the GGB or SEG to the age range 30+ to 65+ reduces the effect of migration; applying both and averaging the results appears to give the smallest error, but this procedure seems very inelegant.

It must be stressed that the simulations analyzed here cover only 98 possible error scenarios, and are unlikely to represent the true distribution of errors in world populations. The summary indicators should therefore only be regarded as a rough guide to how to proceed in the analysis of a real population, and should not be taken as indicative of the likely magnitude of errors in coverage estimates in actual applications.

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