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Research Article

**The impact of immigration under
the defined-benefit pension system:
An analysis incorporating assimilation costs**

Masatoshi Jinnō

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The impact of immigration under the defined-benefit pension system: An analysis incorporating assimilation costs

Masatoshi Jinno¹

Abstract

BACKGROUND

Recently, theoretical studies have started a discussion on how the influx of immigrants affects the finances of the host country.

OBJECTIVE

This paper investigates whether admission of unskilled immigrants, whose children incur assimilation costs in order to become skilled workers, positively influences the net benefits for native residents and immigrants under a defined-benefit pension system (DB system). This paper also compared the results under a DB system with those under a defined-contribution pension system (DC system).

METHODS

This paper theoretically calculates the net benefits for native residents and immigrants under a DB system and compares the values between under a DB system and under a DC system.

RESULTS

The study has three main findings. (1) Under a DB system, native residents do not always become net beneficiaries, even if the government admits an unlimited number of immigrants. This is unlike the analysis under the DC system. (2) The net benefits for native residents caused by permitting a small (large) number of immigrants under the DB system becomes higher (lower) than that under the DC system in certain practical situations. (3) Even if all residents who have the right to vote prefer to admit immigrants, there is a possibility that the net benefits for the native residents may be negative.

CONCLUSIONS

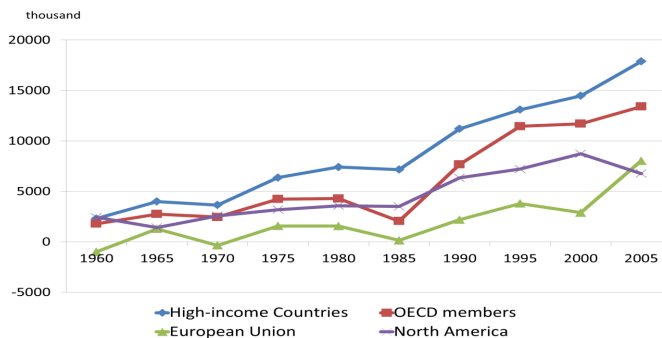
When admitting immigrants, the government must pay attention to the assimilation costs which offspring of immigrants have to pay and the future generation's right to vote because they are the main victims of the loss of benefits caused by the assimilation costs under DB system.

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1 Introduction

Over the past five decades from 1960 to 2010, the stock of immigrants in the world has increased from 71,877,120 in 1960 to 213,316,418 in 2010 according to the data reported by the World Bank. Figure 1 shows the increase in net immigrants among advanced high-income countries including those belonging to the OECD, European Union, and the North American continent during the same periods². However, there is debate among these advanced countries as to whether the admission of immigrants improves the welfare of host countries.

Figure 1: Net total immigrants in advanced economies from 1960 to 2005



Source: World Bank Open Data.

Note: Economies are divided according to 2009 gross national income (GNI) per capita. The high-income group consists of countries with a GNI value of \$12,196 or more.

The empirical literature on the impact of immigration on the finances of the host country is vast, but the findings are mixed. Using US data, Storesletten (2000) found that admitting immigrants brings about a positive but non-significant benefit for the host country. In a later study, Storesletten (2003) also showed that, on average, immigrants in Sweden become net burdens on the host country. By contrast, Schou (2006) showed that not only does admitting immigrants improve the welfare of native residents, the integration or assimilation of immigrants with native residents also leads to a considerable improvement in the government budget. These studies highlight the fact that the admission of immigrants does not always increase net benefits for native residents.

Theoretical studies also discuss how the influx of immigrants affects the finances of

² Economies are divided according to 2009 gross national income (GNI) per capita. The high-income group consists of countries with a GNI value of \$12,196 or more.

the host country. For example, the admission of immigrants increases the number of pension premium payers, which generally helps improve pension finances (Razin and Sadka 1999). It also theoretically showed that even though migrants may be low skilled and the net beneficiaries of a pension system, all existing income (low and high) and age (young and old) groups living at the time of migrants' arrivals in the host country would be better off³. Using an overlapping-generations, small-open-economy model⁴, these authors demonstrated that since the wage rate and contribution rate are constant, admitting unskilled immigrants increases pension premium revenue without causing any losses; hence, native residents always come out as net beneficiaries. Since the defined-contribution pension system (DC system hereafter) also has an income redistribution effect from skilled workers to unskilled workers, when certain conditions are satisfied, unskilled immigrants also become net beneficiaries.

It is important to note, however, that a crucial assumption of Razin and Sadka's (1999) framework is that the offspring of immigrants can all be assimilated and become skilled workers without incurring costs greater than those for native residents. This is not necessarily a realistic assumption.

According to the results presented by the Program for International Student Assessment—a system of international literacy assessments in reading, mathematics, and science—in OECD countries, first-generation immigrant students with foreign-born parents score, on average, 52 points lower than students without an immigrant background (OECD 2010). Riphahn (2002) also showed that even after controlling for certain characteristics, the educational gap between native residents and immigrants in Germany remains significantly large. These results imply that it may be difficult for immigrant offspring to assimilate. Hence, we factored this assimilation difficulty into our model in order to analyze the effects of admitting immigrants.

Krieger (2004) relaxed the assumption that immigrant offspring have the same skill distributions as those of native resident offspring and showed that admitting immigrants does not always increase pension benefits. Although the present study also relaxes this assumption, it differs from that of Krieger (2004) in that we define the difficulty of assimilation in terms of the "assimilation cost"⁵. In general, immigrant offspring require

³ Specifically, native retirees gain, whereas native resident workers are unaffected by admitting immigrants. Thus, it is necessary to transfer the gains from native retirees to native resident workers to improve overall welfare.

⁴ In reality, the small-open-economy assumption may be unsuitable for analyzing the effects of admitting immigrants because host countries are rarely small. Moreover, many econometrics papers, including those by Card (2001) and Zorlu and Hartog (2005), have shown that admitting immigrants does not necessarily decrease the wage rate of native resident workers and that there is little, if any, decrease in the wage rate when this does occur.

⁵ Krieger (2004) showed that pension benefits may change if the assumption is relaxed that the skill distributions

additional time to become skilled workers compared with native resident offspring, as they have to learn the language, customs, and culture of the host country.

Jinno (2011) showed that under the DC system, the net benefits for native residents are not always positive when the offspring of immigrants have to pay the assimilation cost to become skilled; further, whether it becomes positive depends on the number of immigrants. In this paper, the same analysis is carried out under the defined-benefit pension system (DB system hereafter) and the results are compared with those presented by Jinno (2011). This comparison shows that in certain practical situations the net benefits for native residents caused by admitting a small (large) number of immigrants under the DB system becomes higher (lower) those that under the DC system.

DC and DB systems differ as follows. Under a DC system, the pension benefit is an endogenous variable dependent on the total labor income in the next period because the contribution rate is constant for all working generations. By contrast, under the DB system, the contribution rate is an endogenous variable dependent on the total labor income in the next period because the pension benefit is constant for all retirees⁶.

While the differences between pension systems have numerous different effects, the literature has thus far paid insufficient attention to the comparison between the DC and DB systems. Krieger (2003) distinguished between the effects of the admission of immigrants caused by the difference between pension systems under partial equilibrium analysis and investigated the preferred choices of three heterogeneous groups (skilled workers, unskilled workers, and retirees). Wagener (2003) compared the DC and DB systems and showed that the latter is preferred (not preferred) to the former from an *ex post* (*ex ante*) perspective. In addition, Oshio and Yasuoka (2008) compared a DC-funded system, DC system, and DB system under uncertainty to investigate which system is preferable. Finally, using calibration analysis on data from the 2007 Population Projections for Japan, Miyazato (2010) investigated to what degree Japan should maintain a DB public pension system, which offers the benefit of sharing risk, or rather adopt a DC pension system to eliminate intergenerational imbalance. The author showed that a replacement rate of 20% to 30% is required for future generations to achieve the expected return on assets and for the wage growth rate to remain at the same level.

of offspring among native residents and immigrants are the same. However, he did not calculate whether the net benefits for native residents become positive from immigrants through the pension system.

⁶ This is more a Beveridgian system than a Bismarckian one. DB systems fall into two possible classifications: Bismarckian and Beveridgian. Under the Bismarckian system, the individual pension benefit and pension contribution are related to individual earnings during working age. This system is not redistributive. Under the Beveridgian system, which we examine in this paper, there is a proportional contribution from earnings in young age and a flat benefit in old age independent of previous earnings. This system has an intragenerational income redistribution effect. We focus on the Beveridgian system in order to compare our findings with those of Jinno (2011), who investigated the effects of admitting immigrants under a DC regime, which also has an intragenerational income redistribution effect.

However, the effects of admitting immigrants have not been incorporated into these comparative studies, except by Krieger (2003). Even Krieger's (2003) study has a limitation in that it assumes that the offspring of immigrants can assimilate with native residents without incurring additional costs. Because the assimilation costs of the DC and DB systems have not been compared adequately, it is important to investigate the effects of the admission of immigrants on the net benefits for native residents and immigrants in order to consider the assimilation cost immigrant offspring have to pay under the DB system and to compare the results with those presented by Jinno (2011). By meeting this objective, this paper finds that there is no improvement in net benefits for native residents regardless of how many immigrants are admitted under the DB system—even if all residents who have the right to vote are indifferent to or prefer to admit immigrants⁷.

The remainder of this paper is organized as follows. We introduce the model in Section 2 and provide a discussion and concluding remarks in Section 3.

2 The model

We used an overlapping-generations model based on Razin and Sadka (1999) and Jinno (2011) in which individuals live for two periods, namely the working period and the retirement period. During each period, a new generation with a continuum of individuals is born. Individuals are endowed with one unit of time, and they decide whether to become skilled or unskilled workers in the first period. While unskilled workers work for a whole period, skilled workers do so for the remainder of the period after attending school for a certain period of time to be trained. During the second period, they retire from work and consume all the returns from their savings and pension benefits.

In this model, individuals develop their innate abilities in order to become skilled workers. Any individual can become a skilled worker in the first period by attending school for a certain period of time depending on his or her innate ability. Compared with their peers, immigrant offspring require more time to become skilled workers because of the language and culture barriers they need to overcome. Except for this aspect, immigrant offspring are essentially the same as native resident offspring.

The model examines a DB system funded by payroll pension premiums levied on the young working population. This pension system ensures a uniform and constant demogrant for the aged. Here, immigration occurs only in the first period. The subscript n is used to denote native residents and m to denote immigrants.

⁷ Although he focused on the preferred choices of native residents in terms of the number of immigrants, Krieger (2003) paid no attention to the net benefits of admitting immigrants.

2.1 Immigration

In period zero, we assume that the country receives young unskilled immigrants without capital. Although they cannot become skilled workers, their offspring can. However, their offspring need a longer period of education than native-born offspring to become skilled workers. The difference between these education periods represents the cost of assimilation, which we focus on in this study⁸. We assume that the innate abilities of the offspring are distributed similarly and that the fertility rate is identical in both groups in line with the literature (Haupt and Peters 1998; Razin and Sadka 1999; Krieger 2003).

2.2 Individual ability and behavior

There are two levels of work skills: low and high. In this paper, a low-skilled worker is regarded as an unskilled worker and a highly skilled worker as a skilled one. e , which ranges from 0 to 1, represents the innate ability to acquire skills. Individuals can become skilled workers by investing e units of time in school. They will work for the remaining time, denoted by $1 - e$. This assumption implies that a lower e individual is more capable and needs less time to acquire skills.

When becoming skilled workers, the offspring of immigrants have to spend additional time learning the language, culture, and so on. This additional time is the cost of assimilation, φ . This cost is the same value for all immigrant offspring in period zero, but the continuity of the assimilation cost is assumed for only one period. This implies that only the next generation of immigrants has to pay the cost of assimilation⁹.

The cumulative distribution function of ability e is denoted by $G(\cdot)$, where $G(e_i)$, $i = n, m$ refers to the number of individuals with an innate ability parameter that is below or equal to e_i . For simplicity, during the initial analysis of the economy, the number of native individuals born in period zero is normalized to one, that is $G(1) = 1$.

In the first period, each individual decides whether to acquire skills to become a skilled worker, after which he or she then works; bears $1 + a$ offspring (the same for all individ-

⁸ In this paper, the cost of assimilation is limited to the educational costs of the offspring of immigrants who want to become skilled workers. It is reasonable to assume that not only they but also other immigrant individuals have to pay this cost. This includes immigrants as unskilled workers in the first period, the offspring of immigrants to be unskilled workers in the second period, and the offspring of immigrants to be skilled workers in the later period. However, because the expansion of this assumption is outside the scope of this study, this paper assumes only the offspring of immigrants in the first generation to become skilled workers have to pay the cost of assimilation.

⁹ It is easy to expand the assumption and reasonable to consider that the cost of assimilation gradually decreases as the offspring of immigrants are assimilated. Indeed, OECD (2010) showed that second-generation immigrant students lag behind those without an immigrant background by an average of only 33 score points across OECD countries, which is smaller than that of first-generation students. However, the results of expanding this assumption are similar to those obtained in this paper.

uals); consumes a single all-purpose good; and saves for his or her retirement. In the second period (i.e., during retirement), each individual consumes his or her pension and all the returns from savings that were accumulated in the previous period.

In line with the approaches of Razin and Sadka (1999) and Jinno (2011), the present paper also focuses on how migration affects the finances and benefits of a DB system rather than its impact on relative wages. Consequently, we assume a small country with free access to international capital markets. This assumption ensures that the return to capital r is fixed, as determined by international capital markets¹⁰. Further, by assuming a constant returns-to-scale production function, the wage rate w is also fixed independent of the level of migration¹¹.

The income of a native-born individual who decides to acquire skills through schooling is represented by $(1 - e_w)w(1 - \theta)$, where w and θ represent the wage rate per unit of effective labor and the flat social security contribution (tax) rate, respectively. Since the immigrant offspring requires additional time to develop his or her skills (i.e., the assimilation cost), the income of those who work as skilled laborers is represented by $(1 - e_m - \varphi)w(1 - \theta)$. For simplicity, the difference in labor output between skilled and unskilled workers is taken into account by assuming that a skilled worker provides an effective labor supply of one unit for each unit of working time, while an unskilled worker provides only $q < 1$ units of effective labor for each unit of working time. The income of unskilled laborers is represented as $qw(1 - \theta)$.

Individuals choose to become skilled as long as the income of skilled workers is higher than that of unskilled workers. Further, there exists a cutoff level for innate ability e_i^* , where both incomes are equal, that is,

$$(1 - e_n^*)w(1 - \theta) = qw(1 - \theta) \quad (1a)$$

for native individuals, and

$$(1 - e_m^* - \varphi)w(1 - \theta) = qw(1 - \theta) \quad (1b)$$

for immigrants. This indicates that individuals whose innate ability e_i is lower than e_i^* , $i = n, m$ will decide to acquire new skills, whereas those whose innate ability e_i exceeds

¹⁰ The aggregate stock of domestic capital in each period is calculated by adding the aggregate savings the young saved in one period earlier to the net capital investment from abroad. The rate of return from savings thus becomes constant, while all capital is depreciated in one period.

¹¹ The assumption that the wage rate is constant is crucial. As shown by many studies, such as those by Friedberg and Hunt (1995), Card (2001), and Dustmann, Glitz, and Frattini. (2008), the wage elasticity between native residents and immigrants is almost zero. Hence, this assumption may be acceptable. Meanwhile, relaxing the assumption that the wage rate is constant has also been considered important in other studies such as that by Razin and Sadka (2000). However, this relaxation of the constant wage rate assumption has yet to be further examined.

e_i^* , $i = n, m$ will decide not to acquire skills (i.e., remain unskilled). Based on Equations 1a and 1b, we have

$$e_n^* = 1 - q \tag{2a}$$

$$e_m^* = 1 - q - \varphi \tag{2b}$$

Since the productivities of unskilled native and immigrant workers are assumed to be identical, based on 2a and 2b, the relationship between the cutoff levels for innate ability for native and immigrants workers is written as

$$e_m^* = e_n^* - \varphi \tag{3}$$

Equation (3) indicates that the cutoff level for innate ability for immigrant workers, e_m^* that is, the level at which they decide whether to become a skilled or an unskilled worker, is less than that for native individuals. Hence, the number of skilled immigrant workers is found to be less than that of skilled native workers. If $\varphi \geq 1 - q$ holds, all immigrant offspring will become unskilled workers.

2.3 Incomes and utilities of native residents and immigrants

Workers in the t -th period have to pay pension premiums and after that decide on the amounts of savings and consumption, c_t^1 , in the working period. The sum of the returns from savings and constant pension benefits, b , is consumed in the retirement period. There is no bequest. Individuals face the intertemporal budget constraint $c_t^1 + c_t^2/(1+r) = W(e_i)(1-\theta_t) + b/(1+r)$, where c_t^2 is consumption in the retirement period and $W(e_i)$ is the pre-tax income.

The pre-tax income of skilled and unskilled native workers is represented as

$$W(e_n) = \begin{cases} w(1 - e_n) & \text{for } e_n \leq e_n^* \\ qw & \text{for } e_n \geq e_n^* \end{cases} \tag{4a}$$

Meanwhile, the pre-tax income of skilled and unskilled immigrant workers is represented as

$$W(e_m) = \begin{cases} w(1 - e_m - \varphi) & \text{for } e_m \leq e_m^* \\ qw & \text{for } e_m \geq e_m^* \end{cases} \tag{4b}$$

Suppose that the chosen consumption levels in the working and retirement periods are represented by a utility function $u(c_t^1, c_t^2)$ that satisfies the usual assumptions $u_i(\cdot) > 0$, $u_{ii}(\cdot) < 0$, and $u_{ij}(\cdot) > 0$. Maximizing utility with respect to c_t^1 and c_t^2 , subject

to the intertemporal budget constraint, yields an indirect utility function $V_t^i = v((1 - \theta_t)W(e_i), b, r)$, which depends on the pension premium because the other variables are constant. In other words, indirect utility function increases when the pension premium decreases (i.e., when immigrants are admitted) and decreases in the subsequent period because of the labor supply changes described in Section 2.4.

The utility of retired native residents when immigrants are admitted only depends on the sum of returns from savings and pension benefits, which are constant even if immigrants are admitted. Thus, admitting immigrants affects native residents in the working generation and the offspring in the next period but does not affect native retired residents. Although analyzing whether social utility has improved is typical, as the indirect utility function depends on only the pension premium, a decrease in the total pension burden for native residents can improve social utility if an adequate transfer mechanism is adopted. Thus, in line with Jinno (2011), we analyze how admitting immigrants affects the net benefits for native residents and immigrants.

2.4 Labor supply

In this subsection, aggregate labor supply for native residents and immigrants is calculated. The labor supply of skilled native resident workers is denoted by $(1 - e_n)$, while that of unskilled native resident and immigrant workers is denoted by q . The aggregate supply of effective labor in period zero is given by

$$L_0^* = \int_0^{e_n^*} (1 - e)dG + q[1 - G(e_n^*)] + mq, \quad (5)$$

where the first term on the right-hand side of the equation refers to the effective labor supply of skilled native resident workers, while the second term refers to the effective labor supply of unskilled native resident workers. Since it is assumed that immigrants participate in production as unskilled workers in period zero, the effective labor supply of immigrant workers is represented as the third term on the right-hand side of the equation. Further, $E_n^* \equiv \int_0^{e_n^*} (1 - e)dG + q[1 - G(e_n^*)] = (e_n^* - e_n^-)G(e_n^*) + q$ defines the labor force supplied by native residents, where e_n^* denotes the upper bound of the ability parameter of skilled native resident workers and e_n^- the average ability parameter of skilled native resident workers. Thus, $e_n^* > e_n^-$.

The aggregate supply of effective labor in period one is given by

$$L_1^* = (1 + a)[E_n^* + mE_m^*], \quad (6)$$

where $E_m^* \equiv \int_0^{e_m^*} (1 - e - \varphi)dG + q[1 - G(e_m^*)]$ defines the labor force supplied by

offspring whose parents are immigrants. The relationship between E_m^* and E_n^* is $E_m^* < E_n^*$ because e_m^* is less than e_n^* .

By differentiating E_m^* with respect to φ , we get $dE_m^*/d\varphi = -G(e_m^*) < 0$. Thus, an increase in the cost of assimilation decreases the labor force of immigrant offspring. The lower the cost of assimilation is, the larger the labor force in the period after the admission of immigrants.

2.5 The pension system

This paper focuses on a DB system. Retirees receive constant pension benefits b , which are paid from the current pension premiums collected from workers. Thus, $\theta_t w L_t^* = bN_{t-1}$ holds. N_t is the number of t -th generation residents including immigrants. b is a constant value regardless of whether immigrants are admitted or not. By contrast, the equal contribution rate θ_t is endogenously determined to equalize total revenue with total pension benefits. Admitting immigrants affects native residents through the change in the contribution rate in the period in which they arrive.

In period zero, the pension contribution rate becomes $\theta_0^{no} = b/((1+a)E_n^*w)$ when immigrants are not admitted. If immigrants are admitted, the pension contribution rate then becomes $\theta_0^{im} = b/((1+a)(E_n^* + mq)w)$. Deducting θ_0^{im} from θ_0^{no} yields $\theta_0^{no} - \theta_0^{im} = bmq/((1+a)E_n^*(E_n^* + mq)w) > 0$. Thus, admitting immigrants decreases the contribution rate in period zero.

The difference in the total contribution paid by native residents in period zero, P_0 , between no immigrants and some immigrants becomes

$$P_0 = (\theta_0^{no} - \theta_0^{im})wN_0 = \frac{bmq}{(1+a)E_n^*(E_n^* + mq)} > 0 \quad (7)$$

This represents the benefit of admitting immigrants.

Meanwhile, the contribution rate in period one becomes $\theta_1^{no} = b/((1+a)E_n^*w)$, which represents the contribution rate when no immigrants are admitted, while $\theta_1^{im} = b(1+m)/((1+a)(E_n^* + mE_m^*)w)$ represents the contribution rate when immigrants are admitted. Deducting θ_1^{im} from θ_1^{no} yields $\theta_1^{no} - \theta_1^{im} = -bm(E_n^* - E_m^*)/((1+a)E_n^*(E_n^* + mE_m^*)w) < 0$, showing that admitting immigrants increases the contribution rate in period one.

The difference in the total contribution paid by native residents in period one, P_1 , between no immigrants and some immigrants becomes

$$P_1 = (\theta_1^{im} - \theta_1^{no})wN_1 = -\frac{bm(E_n^* - E_m^*)}{E_n^*(E_n^* + mE_m^*)} \quad (8)$$

This represents the burden of admitting immigrants¹².

2.6 Net benefits

Here, we analyze whether admitting immigrants has positive or negative financial impacts on native residents through pension system.

2.6.1 Total net benefits for native residents

The net benefit for native residents, NB_n^{DB} , is the difference between P_0 and the discounted present value of P_1 . This is represented as

$$NB_n^{DB} = \frac{bm[A(E_n^* + mE_m^*) - (E_n^* + mq)(E_n^* - E_m^*)]}{(1+r)E_n^*(E_n^* + mE_m^*)(E_n^* + mq)} \quad (9)$$

where $A \equiv q(1+r)/(1+a)$.

If the assimilation cost is zero, the net benefits for native residents are always positive, namely $NB_n^{DB} = bmq/((1+a)E_n^*(E_n^* + mq)) > 0$. As in the DC system described by Razin and Sadka (1999), native residents become net beneficiaries from admitting immigrants when the assimilation cost is zero.

According to 9, we have

$$NB_n^{DB} \begin{cases} \geq \\ \leq \end{cases} 0, \text{ if } A \begin{cases} \geq \\ \leq \end{cases} B_n^{DB}(m) \equiv \frac{(E_n^* + mq)(E_n^* - E_m^*)}{(E_n^* + mE_m^*)} \quad (10)$$

By differentiating $B_n(m)$ with respect to m , we get $B_n^{DB'}(m) = -E_m^*(E_n^* + mq)(E_n^* - E_m^*)/(E_n^* + mE_m^*)^2 < 0$ and $B_n^{DB''}(m) = 2E_m^{*2}(E_n^* + mq)(E_n^* - E_m^*)/(E_n^* + mE_m^*)^3 > 0$. We also have $B_n^{DB}(0) = E_n^* - E_m^* > B_n^{DB}(\infty) = (q/E_m^*)(E_n^* - E_m^*)$ because the least productive workers among the immigrant population are the unskilled workers who have productivity q , and thus, $E_m^* \geq q$.

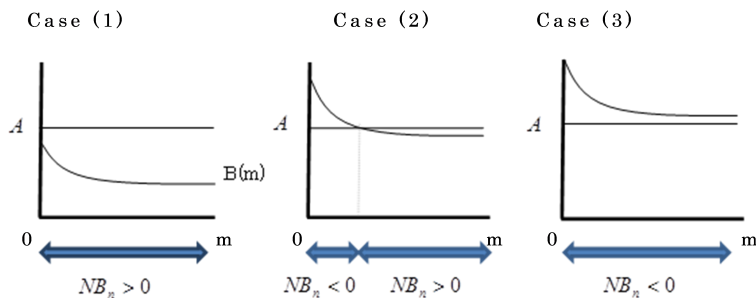
These calculations indicate three relationship scenarios between the number of immigrants and net benefits for immigrants. These relationships, which are described in Figure 2, are: (1) $(A \geq E_n^* - E_m^*)$; (2) $((E_n^* - E_m^*) > A > (q/E_m^*)(E_n^* - E_m^*))$; and (3) $((q/E_m^*)(E_n^* - E_m^*) \geq A)$.

Case (1) $(A \geq E_n^* - E_m^*)$ in Figure 2 illustrates the scenario where the difference between the labor force supplied by native residents and immigrants, $(E_n^* - E_m^*)$, is less

¹² Because a constant assimilation cost is assumed for one period and factor prices (w and r) are fixed, admitting immigrants influences the pension burden for only two periods, namely the period in which immigrants are admitted and the next period. Afterwards, all variables return to their values without any influence from immigrants. Thus, we focus on the changes in the pension burden in these two periods only.

than A . This implies that the assimilation cost is very small and the skill level of immigrants is very high. Hence, admitting immigrants always produces positive net benefits for native residents.

Figure 2: The relationship between the net benefits for native residents and the number of immigrants (1)



Case (2) ($(E_n^* - E_m^*) > A > (q/E_m^*)(E_n^* - E_m^*)$) illustrates the scenario wherein the difference between the labor force supplied by native residents and immigrants, $(E_n^* - E_m^*)$, is larger than A but the rate of skilled workers among the offspring of immigrants is relatively high ($(E_m^*/q) > (1/A)(E_n^* - E_m^*)$). Here, native residents do not become net beneficiaries unless a sufficient number of immigrants are admitted.

Case (3) ($(q/E_m^*)(E_n^* - E_m^*) \geq A$) illustrates the scenario wherein the rate of skilled workers among the offspring of immigrants is relatively low ($(E_m^*/q) \leq (1/A)(E_n^* - E_m^*)$). In this case, native residents do not become net beneficiaries regardless of how many immigrants are admitted. This scenario was not examined by Jinno (2011), who showed that under a DC system in a small open economy, admitting a certain number of immigrants produces positive net benefits for native residents. Our rationale for including Case (3) in this paper is described below.

Under a DC system, although admitting unskilled workers with a constant wage rate increases pension premium revenue linearly, loss is limited and dependent on the constant difference between the productivities of native residents and of immigrant offspring. The net benefits for native residents increase with the number of immigrants. By contrast, under a DB system, native residents do not become net beneficiaries, even if the government admits an unlimited number of immigrants. Benefits (burdens) for native residents from admitting immigrants do not increase (decrease) linearly, but in-

crease (decrease) gradually, namely $\partial P_0/\partial m = bq/[A(E_n^* + mq)^2] > 0$, $\partial^2 P_0/\partial m^2 = -2bq^2/[A(E_n^* + mq)^3] < 0$, $\partial P_0/\partial m = -b(E_n^* - E_m^*)/(E_n^* + mE_m^*)^2 < 0$, and $\partial^2 P_0/\partial m^2 = 2b(E_n^* - E_m^*)E_m^*/(E_n^* + mE_m^*)^3 > 0$. These equations show that whether the marginal benefit exceeds the present value of the marginal burden from admitting immigrants depends on certain parameters and that there is no guarantee native residents will become net beneficiaries even if an unlimited number of immigrants is admitted. In the next subsection, we compare the net benefits for native residents under the DB and DC systems.

2.6.2 Net benefits for native residents under the DB and DC systems

According to Jinno (2011), the net benefits for native residents under the DC system can be calculated as $\frac{m(A(1+m)-(E_n^*-E_m^*))}{A(1+m)}$, which is defined as NB_n^{DC} . This is zero when the number of immigrants is zero and increases to become infinity when an infinite number of immigrants are admitted. However, the net benefits for native residents under the DB system converge to a finite value $\frac{AE_m^*-q(E_n^*-E_m^*)}{q(1+r)E_n^*E_m^*}$, which is positive in Cases (1) and (2) and negative in Case (3), as described in subsection 2.6.1. In any case, the net benefits for native residents under the DC system are higher than those under the DB system when an infinite number of immigrants are admitted.

Next, we compare the net benefits under these two systems when the number of immigrants is very low. By differentiating NB_n^{DB} and NB_n^{DC} with respect to m and evaluating it with $m = 0$, we get $\frac{dNB_n^{DB}}{dm}\Big|_{m=0} = \frac{A-(E_n^*-E_m^*)}{(1+r)E_n^{*2}}$ and $\frac{dNB_n^{DC}}{dm}\Big|_{m=0} = \frac{A-(E_n^*-E_m^*)}{A}$. These calculations lead to

$$\frac{dNB_n^{DB}}{dm}\Big|_{m=0} \begin{matrix} \geq \\ \leq \end{matrix} \frac{dNB_n^{DC}}{dm}\Big|_{m=0} \quad \text{if } q \begin{matrix} \geq \\ \leq \end{matrix} (1+a)E_n^{*2} \quad (11)$$

Equation 11 shows that when the productivity of unskilled workers is sufficiently high, the net benefits for native residents grow under the DB system compared with under the DC system. Note that the condition in equation 11 does not depend on the assimilation cost but rather on the productivity of unskilled workers, which implies the productivity of first-generation immigrants. This implies that the effects of the cost of assimilation on the net benefits for native residents are equal and cancel each other out when the two pension systems are compared.

2.6.3 A numerical simulation

Because this paper assumes realistic parameters, a numerical simulation is now illustrated. According to the assumptions in Razin and Sadka (2000), where the distribution e is

uniform over the interval $[0, 1]$, the productivity of unskilled workers is half that of skilled workers ($q = 0.5$). Further, the population growth rate is 2% ($a = 0.02$) and each period lasts 25 years. Because the assimilation cost is assumed to be 0.1 ($\varphi = 0.1$) and the annual interest rate is 0.001, which implies that $r = (1 + 0.001)^{25} - 1 = 0.025$, we can calculate the variables used in the model: $E_n^* = 0.625$, $E_m^* = 0.580$, $A = 0.502$, and $E_n^{*2} = 0.391$.

A reasonable set of variables generally satisfies the conditions: (1) $A \geq E_n^* - E_m^*$, and (2) $q > (1 + a)E_n^{*2}$. The former condition implies that the reasonable set of variables result in Case (1), where admitting immigrants always produces positive net benefits for native residents. Since the loss in the labor supply of the next period was caused by the limited assimilation cost, the benefit from admitting immigrants surpasses it.

The latter condition implies that while the net benefits for native residents under the DB system are lower than those under the DC system when an infinite number of immigrants is admitted, they are also higher than those under the DC system when the number of immigrants is sufficiently low.

These comparisons between the two systems occur for two reasons. First, while the benefits for native residents under the DC system increase linearly with the number of immigrants, the loss from admitting immigrants is limited (see Jinno 2011). Second, both the benefits and the loss from admitting immigrants are limited under the DB system. Thus, even if the benefits for native residents under the DB system are higher than those under the DC system when the number of immigrants is low, the benefits of admitting additional immigrants under the DC system surpasses the advantage of the DB system when the number of immigrants is very low.

Figure 3 shows the relations between the number of immigrants and the net benefits for native residents using certain set of parameters (please see the Appendix for a detailed analysis).

2.6.4 Total net benefits for immigrants

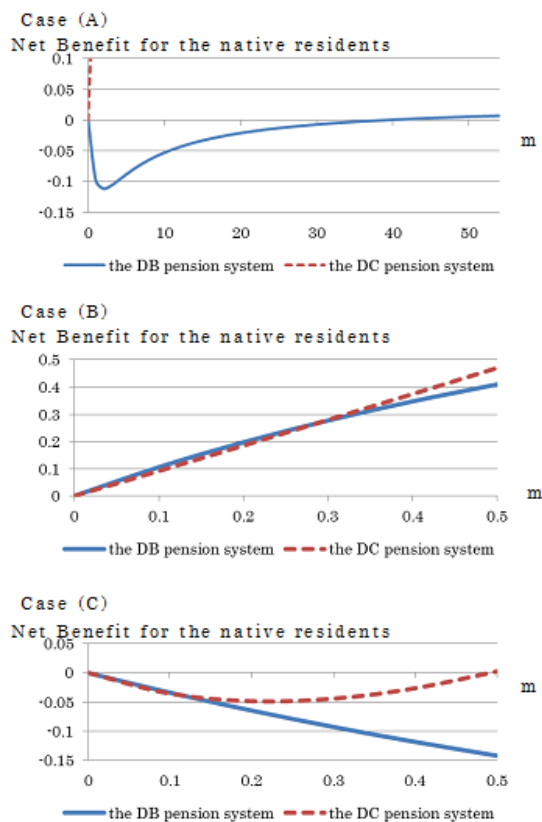
Since retirees receive constant pension benefits \bar{b} , the pension contribution rate becomes $\theta_0^{im} = \frac{b}{(1+a)(E_n^* + mq)w}$. The net benefits for immigrants under a DB system are then defined as

$$NB_m^{DB} = \frac{b}{1+r} - \theta_0 w q = \frac{b}{1+r} \left[1 - \frac{A}{(E_n^* + mq)} \right] \quad (12)$$

Consequently, we get

$$NB_m^{DB} \geq 0, \text{ if } m \geq m_m^{DB} \equiv \frac{1+r}{1+a} - \frac{E_n^*}{q} \quad (13)$$

Figure 3: The relationship between the net benefits for native residents and the number of immigrants (2)¹³



Because $E_n^* > E_m^* \geq q$ holds, if $r < a$ holds, which implies that the economy is in a dynamically inefficient state, the right-hand side of equation 13 becomes negative. Thus, as long as immigration is accepted, immigrants become net beneficiaries under the pension system in this case. Further, even if $r > a$, which means that the economy is dynamically efficient, the right-hand side of equation 13 becomes positive. Thus, a sufficient number

¹³ It is not clear but in all the figures there are some ranges where the net benefits for the native residents under the DB pension become higher than those under the DC system with a small number of immigrants.

of immigrants or the relatively high productivity of skilled native resident workers will make immigrants net beneficiaries in this case.

Regardless of whether there is an assimilation cost, equation 13 determines whether immigrants become net beneficiaries. This implies that the assimilation cost does not affect the net benefits for immigrants. By differentiating NB_m^{DB} with respect to m , we get $\frac{dNB_m^{DB}}{dm} = \frac{b}{(1+a)} \left(\frac{q}{L_0}\right)^2 > 0$ and $\frac{d^2NB_m^{DB}}{dm^2} = -\frac{2b}{(1+a)^2} \left(\frac{q}{L_0}\right)^3 < 0$, showing that an increase in the number of immigrants increases the net benefits for immigrants. However, the rate of increase for these benefits decreases as the number of immigrants increases.

2.7 Discussion on immigration choices

In line with the research by Scholten and Thum (1996), Haupt and Peters (1998), and Krieger (2003), this paper investigates whether native residents are in favor of admitting immigrants by comparing post-immigration lifetime income with pre-immigration lifetime income.

Because pension benefits for retired individuals under a DB system are constant, retirees are indifferent to admitting immigrants. Meanwhile, native residents who are still working prefer to admit immigrants because this decreases the pension premium rate while keeping the wage rate constant. Therefore, under a DB system, retired and working individuals prefer immigration, and thus immigrants are welcomed unrestrictedly.

However, as illustrated by Case (3) in Figure 2, unrestricted immigrant admission does not always produce positive net benefits for native residents, even when all voters, whether retired or working, are indifferent to or in favor of immigration, since this does not reflect the preferences of the offspring of native residents. This population of residents may be against admitting immigrants, as it raises their pension premium rates compared with when no immigrants are admitted. Thus, if $\frac{q}{E_m^*} (E_n^* - E_m^*) \geq A$ holds and the government wants to produce positive net benefits for all native residents, it would choose not to adopt a DB system while continuing to admit immigrants¹⁴.

Although this paper considers skilled and unskilled workers, the relationship between their outputs represents a perfect substitution. Thus, the presented findings are different from those in Krieger (2003) and Ortega (2005), which assumed a complementarity between skilled and unskilled workers¹⁵ and thus that native resident workers are ba-

¹⁴ Current government policies impose taxes on retirees, working individuals, and immigrants in order to compensate for the losses incurred by subsequent generations, who were children when the immigrants were originally admitted. The effects of these policies can be calculated in the same way as in the study by Jinno (2011).

¹⁵ Further, there are other different points between Krieger (2003) and Ortega (2005) and this paper; for example, the offspring in their studies of immigrants are perfectly assimilated and the wage rate is endogenously decided, whereas in this paper the assimilation cost is considered and the wage rate is constant. Krieger (2003) also showed how voting patterns change when a different pension system is adopted, while Ortega (2005), without

sically willing to admit skill-complementary immigration but limit the amount of skill-substitutable immigration. Thus, while the preferred options in these papers are influenced by the conflict among the working generation, those presented herein converge to depend on only the relationship between retired and working generation residents.

However, because the present paper also considers the assimilation cost, it shows the inequality suffered by future generations. This finding suggests that we must pay attention to the future generation's right to vote on whether immigrants are admitted when the DB system is adopted because they are the main victims of the loss of benefits caused by the assimilation costs under DB system.

3 Concluding remarks

This paper investigated whether the admission of immigrants, whose offspring must incur an assimilation cost in order to become skilled workers, produces positive net benefits for both native residents and immigrants under a DB system. We found that while an increase in the number of immigrants directly decreases the burden on the working generation, it indirectly increases the burden on the next generation by raising the pension premium due to the assimilation cost incurred by immigrant offspring. Thus, under the DB system, whether net benefits for native residents are positive depends on certain conditions, and there are cases in which net benefits for native residents do not become positive, regardless of how many immigrants are admitted. This outcome differs vastly from that under a DC system. Moreover, even if the benefits for native residents under the DB system are higher than those under the DC system when the number of immigrants is low under a reasonable set of variables, the benefits of admitting additional immigrants under the DC system surpass the advantage of the DB system when the number of immigrants is very high.

Jinno (2011) showed that under a DC system, a population may not always favor immigration because even if retired individuals are in favor of admitting immigrants, working individuals may still be against it even if their net benefits owing to immigration are positive¹⁶. The present paper showed that under a DB system, a population always favors immigration because retired individuals are indifferent and working individuals are in favor of immigration, even if the net benefits for native residents are negative.

In a real economy, however, the pension system is more likely to be a combination of DC and DB systems. Hence, the appropriate combination of features from these two

considering a pension system but rather considering the expected utilities of children, showed how immigration flows are controlled in the economy when the political effects of immigrants are important and when there is skill upgrading.

¹⁶ If an economy is closed and wage rates are adjusted according to the number of immigrants, preferences among retired and working individuals may change (Krieger 2003).

systems that moves retirees and working individuals towards favoring immigration and produces positive net benefits for all native residents is an issue that needs to be examined further.

It should also be noted that this paper assumed that the wage rates of unskilled workers are constant and that the labor force supplied by native residents and immigrants are perfect substitutes. Some studies have shown that admitting immigrants may have negative, albeit limited, effects on native resident wage rates and that the wage rates of unskilled workers generally decrease as their number increases¹⁷. Hence, it is important to consider the effects on the wage rate of admitting immigrants. In this paper, we also assumed that both native residents and immigrants are employed full-time. The impact of immigration on the unemployment rate is also an issue that needs to be further examined¹⁸.

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¹⁷ Some studies, such as that by Nakamura et al. (2009), have shown a complementary relationship between native and immigrant unskilled workers in Japan.

¹⁸ Kemnitz (2003) examined the impact of admitting immigrants on native residents through unemployment insurance and pension systems.

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Appendix

To analyze in detail how admitting immigrants affects the net benefits for native residents, we carry out a case-by-case analysis. Rearranging equation 10, we obtain

$$NB_n^{DB} = \frac{bm(m(AE_m^* - q(E_n^* - E_m^*)) - E_n^*((E_n^* - E_m^*) - A))}{(1+r)E_n^*(E_n^* + mE_m^*)(E_n^* + mq)} \quad (\text{A-1})$$

We define $m_n^{DB-L} \equiv AE_m^* - q(E_n^* - E_m^*)$ and $m_n^{DB-R} \equiv E_n^*((E_n^* - E_m^*) - A)$. Consequently, we identify four relationship scenarios between the benefits for native residents and the number of immigrants, illustrated as follows:

- Case (A) $m_n^{DB-L} > 0$ and $m_n^{DB-R} > 0$
- Case (B) $m_n^{DB-L} > 0$ and $m_n^{DB-R} < 0$
- Case (C) $m_n^{DB-L} < 0$ and $m_n^{DB-R} > 0$
- Case (D) $m_n^{DB-L} < 0$ and $m_n^{DB-R} < 0$

In Case (A), $m_n^{DB-L} > 0$ implies that $1 + \frac{1+r}{1+a} > \frac{E_n^*}{E_m^*}$, and $m_n^{DB-R} > 0$ implies that $\frac{E_n^*}{E_m^*} > \frac{A}{E_m^*} + 1$. Thus, for Case (A) to be effective, it must satisfy the following condition:

$$1 + \frac{A}{E_m^*} < \frac{E_n^*}{E_m^*} < 1 + \frac{A}{q} \quad (\text{A-2})$$

Because $E_m^* \geq q$, equation A-2 holds. We then have the equation:

$$NB_n^{DB} \geq 0, \text{ if } m \geq m_n^{DB} \equiv \frac{E_n^*((E_n^* - E_m^*) - A)}{(AE_m^* - q(E_n^* - E_m^*))} \quad (\text{A-3})$$

Equation A-3 shows that admitting a sufficient number of immigrants can produce positive net benefits for native residents. By differentiating m_n^{DB} with respect to φ , we get

$$\frac{dm_n^{DB}}{d\varphi} = \frac{E_n^*A(A+q-E_n^*)}{(m_n^{DB-L})^2} \frac{dE_m^*}{d\varphi} = -\frac{E_n^*A(A+q-E_n^*)}{(m_n^{DB-L})^2} G(e_m^*) \quad (\text{A-4})$$

By substituting the definition of E_m^* into equation A-4, we get

$$\frac{dm_n^{DB}}{d\varphi} = \frac{((e_n^* - e_n^-)G(e_n^*) + q)A((e_n^* - e_n^-)G(e_n^*) - A)}{(m_n^{DB-L})^2} G(e_m^*) \quad (\text{A-5})$$

where $(e_n^* - e_n^-)G(e_n^*)$ is the number of skilled workers multiplied by the difference between the upper bound of the ability parameter and the average ability parameter for skilled native resident workers. $(e_n^* - e_n^-)G(e_n^*)$ also represents the total excess labor supply of skilled workers over the productivity of unskilled workers among native residents.

Thus, if the total excess labor supply of skilled workers is greater than A , the host country needs to admit more immigrants to produce positive net benefits for native residents as the cost of assimilation increases. This is necessary because a higher value of $(e_n^* - e_n^-)G(e_n^*)$ implies a relatively high productivity of skilled native resident workers, and admitting unskilled immigrant workers lowers this productivity. Consequently, to produce positive net benefits for native residents, a sufficient number of immigrants must be admitted.

We then compare the thresholds in the number of immigrants under the DB and DC systems that render the net benefits for native residents positive. Since $m_n^{DB-L} > 0$ and $m_n^{DB-R} > 0$ hold, differentiating equation 10 with respect to m yields

$$\frac{dNB_n^{DB}}{dm} = \frac{b}{(1+r)E_n^*} \frac{M + 2m_n^{DB-L}E_n^*m - (E_n^*)^2m_n^{DB-R}}{(E_n^* + mE_m^*)^2(E_n^* + mq)^2}, \quad (A-6)$$

$$\text{where } M = (m_n^{DB-L}(q + E_m^*) + m_n^{DB-R}E_m^*q)m^2$$

Evaluating equation A-6 at zero immigration, we have

$$\left. \frac{dNB_n^{DB}}{dm} \right|_{m=0} = -\frac{b}{(1+r)(E_n^*)^2} m_n^{DB-R} < 0 \quad (A-7)$$

Equation A-7 shows that admitting immigrants decreases the net benefits for native residents at first. However, we also have

$$dNB_n^{DB} \Big|_{m=0} = 0 \quad (A-8a)$$

$$dNB_n^{DB} \Big|_{m=\infty} = -\frac{b}{q(1+r)E_m^*E_n^*} m_n^{DB-L} > 0 \quad (A-8b)$$

According to equations A-6, A-7, and A-8, while admitting immigrants produces negative net benefits for native residents at first, the rate of the decrease in net benefits gradually diminishes until this burden is exceeded by the benefits of admitting immigrants. As a result, the net benefits for native residents become positive and converge to $\frac{b}{q(1+r)E_m^*E_n^*} m_n^{DB-L}$.

Jinno (2011) showed that under the DC system, a certain number of immigrants must be admitted in order to produce positive net benefits for native residents. This threshold

level is defined as $m_n^{DC} \equiv \frac{E_n^* - E_m^*}{A} - 1$. The difference between m_n^{DB} with m_n^{DC} is shown as

$$m_n^{DB} - m_n^{DC} = \frac{(q + A)(E_n^* - E_m^*)((E_n^* - E_m^*) - A)}{A(AE_m^* - q(E_n^* - E_m^*))} > 0 \quad (\text{A-9})$$

where the assimilation cost is the same in both pension systems. Thus, $m_n^{DB} > m_n^{DC}$. This implies that the required number of immigrants for native resident benefits to be positive under a DB system is higher than that under a DC system.

These calculations show that the required number of immigrants for native resident benefits to be positive under a DB system is higher than that under a DC system. This is because even though an increase in the number of immigrants increases pension benefits linearly under a DC system, a similar increase under a DB system increases pension benefits, but at a diminishing rate.

In Case (B), equation A-1 shows that native residents always become net beneficiaries by admitting immigrants. Since in this case $m_n^{DB-L} > 0$ and $m_n^{DB-R} < 0$, we can get only $E_n^* - E_m^* < A$. This implies that the difference between native resident and immigrant productivities, which depend on the assimilation cost, is not sufficiently high.

In Case (C), equation A-1 shows that the net benefits for native residents of admitting immigrants are always negative. In this scenario, $A < \frac{q(E_n^* - E_m^*)}{E_m^*}$ holds. This implies that the difference between native and immigrant productivities is sufficiently high, which also means that the assimilation cost is very high.

For Case (D) to be realized, we must satisfy the condition $E_n^* - E_m^* < A < \frac{q(E_n^* - E_m^*)}{E_m^*}$. Since $E_m^* \geq q$ holds, however, Case (D) cannot be realized.

The varying net benefits for native residents illustrated by these four cases are summarized as follows. If the assimilation cost for immigrant offspring is very high, native residents will never be net beneficiaries, regardless of how many immigrants are admitted. Meanwhile, if the assimilation cost is very low, native residents will become net beneficiaries regardless of how few immigrants are admitted. When the assimilation cost is within a certain range, a certain number of immigrants must be admitted in order to produce positive net benefits.

According to the numerical analysis using a reasonable set of variables presented above, we have $m_n^{DB-L} = 0.269$ and $m_n^{DB-R} = -0.286$. Thus, this set of parameters results in Case (B), where the net benefits for native residents are always positive. Thus, the sets of reasonable parameters tend to result in Case (B).

However, when the assimilation cost is high and the productivity of unskilled workers is low, the situation results in Cases (A) or (C). For example, if the assimilation cost is 0.45 ($\varphi = 0.45$), the productivity of unskilled workers is 0.2 ($q = 0.2$), and the rest of the parameters are the same as stated above, this set of parameters results in Case (A);

$m_n^{DB-L} = 0.001$ and $m_n^{DB-R} = 0.030$. However, when the productivity of unskilled workers is the same as in Case (A) ($q = 0.2$), the assimilation cost is a little higher ($\varphi = 0.6$), and the rest of the parameters are the same as above, this set of parameters results in Case (C); $m_n^{DB-L} = -0.016$ and $m_n^{DB-R} = 0.051$. These results are illustrated in Figure 3, where the net benefits for native residents under the DC system are also included with the same parameters.