DEMOGRAPHIC RESEARCH

Formal Relationship

Variable-r in sex ratios: Formulas in honor of Jim Vaupel

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Contents

Variable-r in sex ratios: Formulas in honor of Jim Vaupel

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Abstract

BACKGROUND
Two seminal studies in the 1980s, by Preston and Coale (1982) and Arthur and Vaupel (1984), generalized the Lotka equations developing the variable-r methods.

OBJECTIVE
Time changes in sex ratios (males:females) are studied from the perspective of the variable-r method to estimate the contributions of fertility, mortality, and net-migration.

METHODS
The time change in sex ratios can be calculated as a comparison of the growth rates of the sexes. The difference is then decomposed into population composition, and the age-specific components of fertility, mortality, and net migration. Thirteen countries with long historical demographic series are used to illustrate the time trends in sex ratios.

RESULTS
Most countries are moving towards a greater number of males per females. The greatest changes were observed in Norway and Sweden, with males catching up with females at older ages due to survival. Meanwhile, the sex differential in net-migration flows explains the decline in sex ratios in Spain and Australia and the increase in the United States.

CONCLUSIONS
Our results shed light on the debate on female–male population imbalances and how fertility, mortality, and net migration contribute to these disparities.

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CONTRIBUTION
Vaupel and Canudas-Romo (2003) wrote a paper in honor of Vaupel's mentor Nathan Keyfitz on his 90th anniversary. The paper focused on a common topic of interest of all three researchers, namely mathematical demography. In honor of the legacy of the work of Jim Vaupel, we revive that idea with a collection of mathematical derivations that are simple, yet powerful in their demographic interpretation.

1. Introduction

In 2022, globally there were more males (50.3%) than females, yielding a sex ratio of 1.01, or 101 males for every 100 females (UN 2022). While informative, this overall sex ratio obscures considerable variation in age-specific sex ratios. For example, newborns are more likely to be male, while worldwide at ages 65 and older there are more females (55.7%) than males (UN 2022). The entire collection of varying age-specific sex ratios determines the overall sex ratio of a population. Of this collection, Coale (1991) wrote: "differences in the masculinity ratio [or sex ratio] in a population depend on three factors: differences in the ratio at birth, differences in gains and losses through migration, and differences in mortality." This paper develops and applies several formulas for decomposing temporal changes in male:female sex ratios into the demographic components identified by Coale. We also develop a new formula that incorporates the variable-r method (Arthur and Vaupel 1984; Preston and Coale 1982). Since sex ratios comprise a large field of study, these formulas can provide researchers with new tools to elucidate the demographic determinants of sex ratios. It is our hope that this contribution will also honor the recent passing of the eminent mathematical demographer Jim Vaupel, one of the initial developers of the variable-r method.

1.1 Background and relevance

In this section we provide some background on sex ratios and our approach to understanding demographic contributors. First, we provide a brief overview of the many important applications of the demographic male:female sex ratio. We next briefly overview how Coale's three components of the sex ratio at birth and differential survival and migration may influence sex ratios. We then provide a simple overview of the variable-r method that we incorporate and some of the countries we focus on.

Understanding the demographic determinants of changes in the male:female sex ratio can inform policy, industry, and science. The male:female sex ratio comprises a

large field of study (Schacht et al. 2022). This key demographic metric has informed public policy issues such as labor markets, marriage markets and family formation, health, and crime (Angrist 2002; Schacht, Rauch, and Borgerhoff Mulder 2014; Zhou et al. 2011). For example, uneven sex ratios may generate challenges for individuals desiring to start a family (Diamond-Smith and Rudolph 2018). They may also influence individual behavioral choices, such as partner selection (Oser et al. 2017), as well as economic behavior and financial decision-making (Griskevicius et al. 2012). Researchers have long hypothesized that sex ratios are linked to violence, though the direction of the association is debated (Schacht et al. 2014).

Sex ratios have further applications beyond human demography. They are important for the conservation of species, the viability of ecosystems, and maximizing livestock yields (Beddington 1974; Donald 2007). They are crucial for scientific fields such as evolutionary biology – Charles Darwin (1871) devoted considerable time to the topic.

1.2 Demographic components of changing sex ratios

Given the importance attributed to sex ratios, it is natural to ask why they change over time. The three components of sex ratios mentioned by Coale (1991) suggest a decomposition into birth, survival, and migration components. Each is important and deserves formal integration within demographic models.

The first component is the sex ratio at birth, which is usually male-biased (a common scientific term for a male:female sex ratio larger than 1 – see Caselli et al. 2006; Schacht et al. 2022; Schacht, Rauch, and Borgerhoff Mulder 2014). A conventional estimate of the average sex ratio at birth is 1.05, which has long proved useful for demographic work (e.g., Hollingshaus et al. 2019; Keyfitz 1971). However, there is considerable variation, reflecting social mechanisms such as social preferences for male offspring (Bongaarts 2013) and biological mechanisms related to differential embryo frailty or even sex determination (Bruckner, Catalano, and Ahern 2010). Time trends in the sex ratio at birth changed drastically in the last quarter of the 20th century, particularly in the son-preference populations of China and India (Hull 1990; Jha et al. 2006). By 2017, sex ratios at birth ranged from 103 males per 100 females in sub-Saharan Africa to 113.3 in East Asia (Chao et al. 2019).

The second component in sex ratios is the difference in mortality between the sexes. Excess mortality of males to females is observed in historical data (Beltrán-Sánchez, Finch, and Crimmins 2015), in selected groups of the population (Luy 2003), in times of extreme stress (Zarulli et al. 2018), and in current populations (Zarulli, Kashnitsky, and Vaupel 2021). Thus, as cohorts age, a male-biased infant population eventually becomes female-biased in old age (Hollingshaus et al. 2019). At extremely old ages the disparity

between counts of females and males is so wide that the sex ratio is sometimes inversely calculated; indeed, evidence exists for centenarian female:male sex ratios ranging from 5 to 7 females for every male age 100 years and older in low mortality countries, though there are some cases of ratios close to parity (Caselli et al. 2006; Robine and Caselli 2005).

While this general trend of a decreasing sex ratio with age may be true globally, migration can perturb the trend for smaller geographies. Migration is often patterned by age and sex (Raymer et al. 2018; Rogers, Little, and Raymer 2010), affecting population composition in both origin and destination countries. For example, research suggests that since the 1980s, migration to the United States has tended to favor males (Gordon 2005), while Australia shows more female migrants (Raymer et al. 2018). By comparison, Sweden has had little in-migration historically, but has recently experienced considerable in-migration of young male refugees (McAuliffe and Triandafyllidou 2022).

The three components are often interrelated. For example, a low sex ratio at birth might indicate higher levels of miscarriage among frail male embryos, which might shrink or expand male–female mortality differentials by producing either "culled" or "damaged" cohorts (Catalano and Bruckner 2006). Since migrants are often healthier than non-migrants (Cortes-Bergoderi et al. 2013), migration patterns can affect the aggregate mortality rates by changing population composition. Patterns in mortality and sex ratios have been used to obtain indirect estimates of undocumented migration (Bean King, and Passel 1983). Furthermore, migration can indirectly affect the sex ratio at birth, as evidenced by son preference among mainland Chinese migrants to Hong Kong (Basten and Verropoulou 2013).

1.3 Variable-r decomposition

We aim to help lay a demographic foundation for testing hypotheses of how age-specific dynamics might interact to produce an overall sex ratio for different countries. Solid demographic data and decomposition techniques can provide a straightforward way to describe how these components of change have affected changing sex ratios at different ages. In our analysis we study age-specific sex ratios, consistently defined as males to females, along all the age-span.

While we consider several decomposition methods, our emphasis is on the variable-r approach which incorporates historical cohort demographic information into period measures. The variable-r method was initially developed by Preston and Coale (1982) and Arthur and Vaupel (1984), who defined the mathematical relationships existing between age-specific population growth and age-specific rates of fertility, mortality, and migration. Many demographic analyses have found it useful for understanding the factors

contributing to population aging and disentangling the relative contributions of demographic components (fertility, mortality, and migration) to changes in populations' age patterns (Horiuchi 1991; Horiuchi and Preston 1988; Lee and Zhou 2017; Murphy 2017; Preston, Himes, and Eggers 1989; Preston and Stokes 2012; Preston and Vierboom 2021).

In this study we adapt the variable-r method to assess temporal changes in age-specific and overall sex ratios for 13 countries: Australia, Denmark, England and Wales, Italy, Finland, France, Norway, Scotland, Spain, Sweden, Switzerland, the Netherlands, and the United States. For each, we investigate the contributions of time changes in birth counts, and changes between survival and net-migration probabilities. As this produces a considerable amount of demographic analysis, we provide a more in-depth examination of four countries: Australia, Sweden, the Netherlands, and the United States. Within these low-mortality countries, Australia and Sweden have high survival rates and the United States and females in the Netherlands comparatively low survival rates (Ho and Hendi 2018). In addition, Australia, the Netherlands, and the United States are historically high-migration countries, whereas Sweden has only recently experienced high levels of immigration (McAuliffe and Triandafyllidou 2022).

2. Data

Our analyses use data from the Human Mortality Database (HMD 2023). The HMD contains high quality historical data that combine vital statistics and census counts or official population estimates, and standard methods applied across all populations over time to ensure comparability (Wilmoth et al. 2021). Age-specific counts of population and births, and age-specific death rates were obtained from the HMD for the years 1909 to 2020. The population counts correspond to January 1st of the respective year; thus COVID-19 is not included in the analysis for 2020. We present the results for 13 selected populations: Australia, Denmark, England and Wales, Italy, Finland, France, Norway, Scotland, Spain, Sweden, Switzerland, the Netherlands, and the United States. The England and Wales and French series include total population comprising both civilian and military populations and deaths. For the United States the HMD allowed us to build a truncated historical series from 1933 to 2020. To increase the US historical series from 1909 to 1932, we relied on birth data from the Centers for Disease Control and Prevention (CDC 2020), mortality data from the Social Security Administration (Bell and Miller 2005), and population estimates from the US Census Bureau (US Census 2016). This additional information was required to explore the drivers of the sex ratio in the United States up to age 100 (details found in Appendix 1). That is, the results for the United States are truncated. However, the missing age groups did not disrupt our findings

considerably given that in 2020 the American sex ratio including ages from birth to age 100 was 0.976, the same value as when including all counts up to the last age attained in the population. For Australia, the HMD historical series only allowed exploring changes in sex ratios between 2010 and 2020 up to age 88 (in 2020 the Australian sex ratios up to that age versus for the entire population were 0.991 and 0.985, respectively). Similarly, in Spain the HMD historical series only allowed the comparisons up to age 100 (in 2020 the Spanish sex ratios up to that age versus for the entire population were 0.962 and 0.961, respectively).

3. Methods

Let $P_F(t)$ and $P_M(t)$ correspond to the population counts of females and males at time t, respectively. The population sex ratio is defined as the number of males to females or the ratio

$$SR(t) = \frac{P_M(t)}{P_F(t)} . \tag{1}$$

3.1 Age-specific sex ratios

The sex ratio can be studied as an average of age-specific sex ratios, or $SR(x,t) = \frac{P_M(x,t)}{P_F(x,t)}$ at age x and time t, weighted by the proportion of females at each age (Guillot 2002). Following the variable-r method, the counts of females and males at age x and time t in the age-specific sex ratios are separated into: (1) the product of babies born $t\text{-}x$ years ago, (2) multiplied by the cohort probability of surviving, and (3) the cohort probability net migration to age x. For example, for females at age x and time t we have $P_F(x,t) = B_F(t-x)\, \ell_F(x,t)\, m_F(x,t)$, where $B_F(t-x)$ are the female babies born in year $t\text{-}x$, and $\ell_F(x,t)$ and $m_F(x,t)$ are the cohort probabilities of survival and net migration from birth in year $t\text{-}x$ to age x in year t, respectively. The age-specific sex ratios are then written into three ratios

$$SR(x,t) = R_B(t-x)\, R_\ell(x,t)\, R_m(x,t), \tag{2}$$

where the ratio of births is defined as $R_B(t-x) = \frac{B_M(t-x)}{B_F(t-x)}$, the ratio of survival as $R_\ell(x,t) = \frac{\ell_M(x,t)}{\ell_F(x,t)}$, and the ratio of net migration as $R_m(x,t) = \frac{m_M(x,t)}{m_F(x,t)}$.

As shown in Equation (2), the change over time in the age-specific sex ratios will arise from changes in the ratios of births, survival, and/or net migration. Similar elements can be found for the change over time in the overall sex ratio in Equation (1).

3.2 Decomposing the change in the sex ratio over time

To decompose the change in the sex ratio in Equation (1), we use a simple and succinct notation of a dot on top of a variable to denote the derivative with respect to time (Vaupel and Canudas-Romo 2002, 2003). The change over time in the sex ratio is

$$\dot{SR}(t) = SR(t)[r_M(t) - r_F(t)], \tag{3}$$

where $r_s(t) = \frac{\dot{P}_s(t)}{P_s(t)}$ is the population growth rate for sex s and time t. Equation (3) shows that depending on whether female or male population growth is higher, the sex ratio will decline $[r_M(t) < r_F(t)]$ or increase $[r_M(t) > r_F(t)]$.

Population growth rates can be written as the average of the sex- and age-specific growth rates, denoted as $r_s(x, t)$ for sex s, age x, and time t, weighted by the proportion of the population at that age x, $c_s(x, t) = \frac{P_s(x,t)}{P_s(t)}$ (Horiuchi 1991; Canudas-Romo, Shen, and Payne 2022). Substituting that average for the female and male growth rates in Equation (3) we have

$$\dot{SR}(t) = SR(t) \int_0^\omega [r_M(x, t) c_M(x, t) - r_F(x, t) c_F(x, t)] dx. \tag{4}$$

The age-specific growth rates $r_s(x, t)$ and population composition $c_s(x, t)$ for sex s, age x, and time t combine to determine greater or smaller age-specific contributions to the change in sex ratio in Equation (4). The proportion of the population decreases (in Sweden in 2020 at age 100 there were 0.02% females) or increases (at age 50 there were 1.23% of Swedish females) the contribution of the age-specific growth rates to the change in the overall sex ratio. However, it is also the difference in population composition between females and males which determines the contribution to the sex ratio. Following Kitagawa's (1955) decomposition for a difference in rates as the $[r_M(t) - r_F(t)]$ in Equation (3) allows us to determine the contribution of the population composition and the population growth rate in Equation (4) as

$$\dot{SR}(t) = SR(t) \left(\int_0^\omega [r_M(x,t) - r_F(x,t)] \left[\frac{c_M(x,t) + c_F(x,t)}{2} \right] dx \right.$$

$$\left. + \int_0^\omega \left[\frac{r_M(x,t) + r_F(x,t)}{2} \right] [c_M(x,t) - c_F(x,t)] dx \right), \tag{5}$$

where the first term on the right corresponds to the contribution of the age-specific growth rates and the second term to the contribution of the population composition.

3.3 Variable-r in the change in sex ratios

Following the work on variable-r by Preston and Coale (1982) and Arthur and Vaupel (1984), the age-specific growth rates in Equation (4) can be further decomposed to include cohort information on growth rate at birth, and the change over time in the probability of survival and net migration between birth and age x (Horiuchi and Preston 1988):

$$r(x,t) = r_B(t-x) + r_\ell(x,t) + r_m(x,t), \tag{6}$$

where $r_B(t-x) = \frac{\dot{B}(t-x)}{B(t-x)}$ denotes the growth rate at birth between the *t-x* and *t-x+1* cohorts and $r_\ell(x,t) = \frac{\dot{\ell}(x,t)}{\ell(x,t)}$ and $r_m(x,t) = \frac{\dot{m}(x,t)}{m(x,t)}$ are the terms that compare the change in probabilities of survival and net migration for the birth cohorts *t-x* and *t-x+1* from birth to age *x*. According to Preston and Stokes (2012), the net-migration effect can be calculated as the residual term of the other three components in Equation (6), as opposed to including country-specific data on actual migration flows. The same procedure is followed here. Since the rest of the information needed to calculate Equation (6) – births, mortality, and population – is available in our main source of information, namely the Human Mortality Database, the estimated migration results will (1) have similar accuracy to the rest of the HMD data, and (2) ensure that our results, as presented in Equation (6), complement each other and return an exact result. Comparison of these migration estimates with United Nations (2022) information shows consistency, although for some countries the pattern is unusual (results not shown). The variable-r components allow us to understand how these three demographic elements shaped the population of females and males observed in the sex ratios.

Substituting Equation (6) in Equation (4) for females and males returns

$$\dot{SR}(t) = SR(t) \int_0^\omega \left[r_{B,M}(t-x)c_M(x,t) - r_{B,F}(t-x)c_F(x,t) \right] dx$$

$$+ SR(t) \int_0^\omega \left[r_{\ell,M}(x,t)c_M(x,t) - r_{\ell,F}(x,t)c_F(x,t) \right] dx$$

$$+ SR(t) \int_0^\omega \left[r_{m,M}(x,t)c_M(x,t) - r_{m,F}(x,t)c_F(x,t) \right] dx. \tag{7}$$

As in Equation (4), the weighting function is the structure of the population $c_F(x,t)$ for females and $c_M(x,t)$ for males. Equation (7) corresponds to the decomposition of the change in the sex ratio into the female and male shares of their growth rate at birth as well as past survival and net migration. This equation presents both a decomposition of the change over time in the sex ratio at the age-aggregated level and the age-specific contribution of the sex ratio at birth, and changes in survival and net migration. As opposed to section 3.2, the results presenting Equation (7) combine the population composition with the three components (growth at birth and changes in survival and net migration). Furthermore, in the Appendix we present for each of these components calculations for females and males separately (see Figure A-1). For example, for the growth rate at birth, the first term in Equation (7) is the female component $SR(t) \int_0^\omega r_{B,F}(t-x)c_F(x,t)dx$ and $SR(t) \int_0^\omega r_{B,M}(t-x)c_M(x,t)dx$ is the male component, and it is similar for the other terms.

In the presentation of results in the following section we first present the overall sex ratio in 2010 to 2020, as in Equation (1), and then the three age-specific sex ratios as in Equation (2), in 2020 for selected countries. There follows the decomposition in Equation (5) of the change over time in the sex ratio into growth rate and population composition, as well as the three components using variable-r in Equation (7). All the calculations in this study (estimation details in Appendix 2) were carried out using 'R' software (R Core Team 2021).

4. Results

As observed in Figure 1, between 2010 and 2020, males began to outnumber females in Norway and Sweden, moving from under 100 to over 101 males per 100 females. The change in the sex ratio was more marked in these two countries and Switzerland than in the rest of the analyzed countries. This increasing trend in the sex ratio was observed in most countries; however, Australia, France, and Spain experienced a decline in the ratio, which was particularly stressed in the latter country. However, even the stagnation with a minor decline in France should be further decomposed into its constituent parts to fully

understand if there were actually no changes in the components, or if they counter-balanced each other.

**Figure 1:   Sex ratio for selected HMD countries in 2010 and 2020**
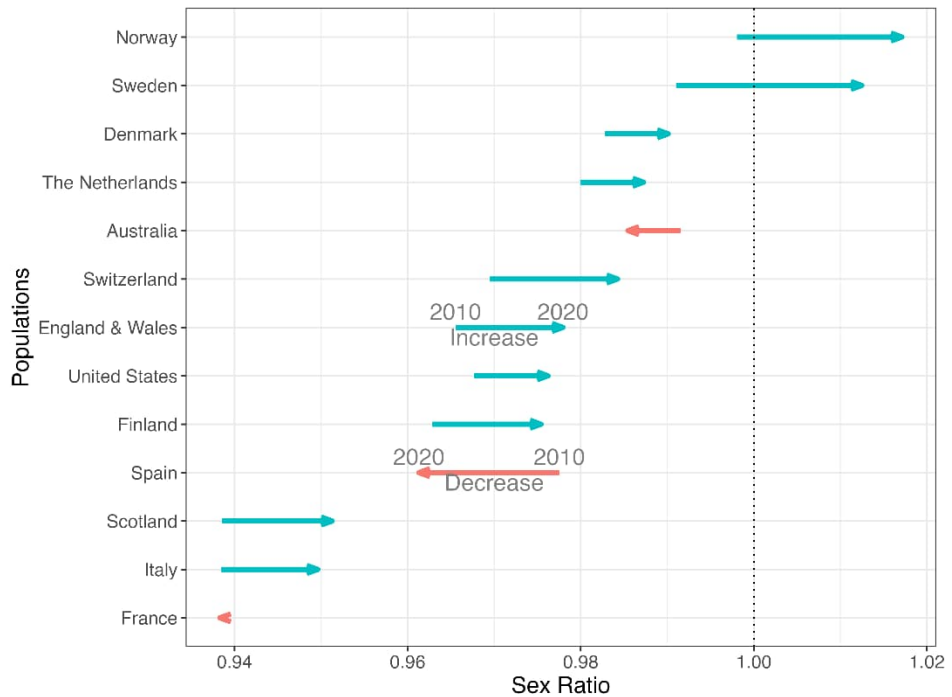
Figure 2 shows the male-to-female, age-specific sex ratios as the product of three ratios (see Equation (2)): the sex ratio at birth, the ratio of survival, and the net-migration ratio, for each of the cohorts present in 2020 in Australia, the Netherlands, Sweden, and the United States. Minor fluctuations – between 1.04 and 1.08 – in sex ratio at birth are observed among the sex ratios by age, with the United States (with the largest population of the four) showing the smoothest of all age patterns. The sex ratio by age starts at the level of the sex ratios at birth, increasing slightly in Australia and considerably in Sweden in the age groups 18–26 due to changes in the ratio of net migration, and declining in all the countries thereafter. However, the crossing of equal numbers of males and females at

a ratio of 1 occurs as early as age 27 in Australia and age 38 in the Netherlands, at age 40 in the United States, and not until age 62 in Sweden. Due to the lower survivorship of males than females, the sex ratios for each cohort fall sharply at older ages. A net-migration ratio that favors females intensifies the lower sex ratio for each of the cohorts in Australia, the Netherlands, and the United States. However, for older cohorts in Australia the higher-than-1 ratio of male:female net migration has a counterbalancing effect.

Figure 2: **Sex ratio by age as a product of the three ratios including cohort information on birth counts, survival, and net migration. Australia, Sweden, the Netherlands, and the United States, 2020**
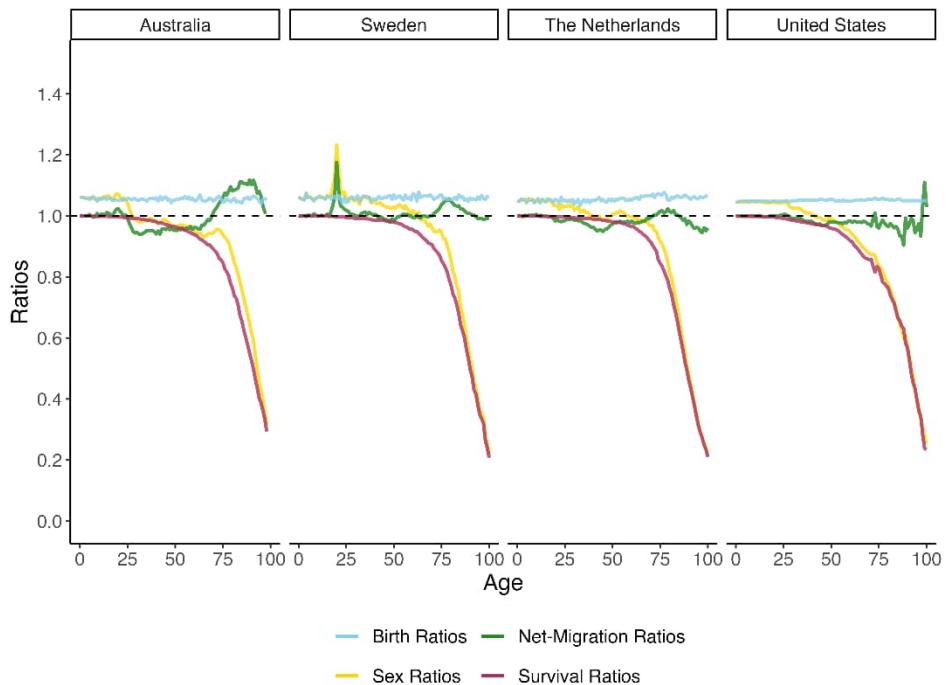
Source: Author's calculations applying Equation (2). Based on data from HMD (2023), CDC (2020), Bell and Miller (2005), and the US Census (2016).
Note: Values at each age x correspond to the ratio (male to female) of birth counts at time 2020-x, as well as cohort survival and net migration between birth and age x. Values above 1 correspond to a greater number for males than for females and values below 1 the opposite.

Table 1 presents the sex ratios in 2010 and 2020, their difference, and the contribution of growth rates and population composition to that difference as in Equation (5). Of the two components that explain the change over time in the sex ratio, the population composition is negative for every single country, meaning that this component contributes to the decline in the sex ratio. This can be explained by the lower proportions of males than females, particularly at older ages. However, in all countries the growth rate contribution opposes the declining trend, with greater average age-specific growth rates for males than females except for Australia and Spain, where negative values are found for this component. Among the countries with opposing trends in the two components, only France has a lower growth-rate contribution than the declining effect of the population composition, and thus has a smaller sex ratio in 2020 than in 2010.

Table 1: **Sex ratios in 2010 and 2020 and decomposition of the change into differences in growth rates and differences in population composition between females and males. Selected HMD countries**

Country	Sex ratio			Difference in growth rates	Difference in population composition
	2010	2020	Change		
Norway	99.8	101.7	1.9	2.3	−0.4
Sweden	99.1	101.2	2.1	2.5	−0.3
Denmark	98.3	99.0	0.7	1.5	−0.8
The Netherlands	98.0	98.7	0.7	1.7	−1.0
Australia	99.2	98.5	−0.6	−0.4	−0.3
Switzerland	96.9	98.4	1.5	2.3	−0.8
England & Wales	96.6	97.8	1.3	1.9	−0.6
United States	96.8	97.6	0.9	1.7	−0.9
Finland	96.3	97.5	1.3	3.0	−1.7
Spain	97.8	96.1	−1.6	−0.2	−1.5
Scotland	93.9	95.1	1.3	2.1	−0.8
Italy	93.8	95.0	1.2	2.8	−1.6
France	93.9	93.8	−0.1	1.2	−1.3

Source: Author's calculations applying Equation (5). Based on data from HMD (2023).
Note: All values multiplied by 100. Ordered from the lowest to the highest sex ratio in 2020. Positive values in the components (difference in growth rates and difference in population composition) correspond to elements that contribute to the increase in the sex ratio, while negative values contribute to the decline in the sex ratio.

Table 2 presents results using Equation (7) to illustrate how the components of change have accumulated over history to affect the overall sex ratio in selected populations, with additional detail found in Figure A-2 in Appendix 3. The strong increase in growth rate at birth for Australian females and males, and to a lesser extent for the United States, is in contrast to the rest of the countries. The starkest contrast is with the negative growth at birth observed for Italy, Scotland, Spain, Finland, Denmark and, to a lesser extent, the Netherlands. By contrast, Italy, Spain, and Finland experienced

the greatest progress in survival, and Australia and the United States the least. Regarding net migration, the largest changes are observed in Norway, Sweden, Switzerland, and Scotland. France is the only country with declining rates of net migration.

However, it is not the change in these three components that contributes to the sex ratio, but the extent to which these changes differ between the sexes. The largest impacts come from differential changes in mortality, with male survival probabilities improving more than female survival probabilities in every analyzed country (seen in the positive values of the difference in survival). This is the most important component in all countries except Australia and Spain, where it is the difference between the sexes in net migration, and Sweden and the United States, where the values match those of the survival differences. The reason for such outcomes is, for the former two countries, the greater inflow of females, and for Sweden and the United States, bigger changes in the male probabilities of net migration. Table 2 further shows that, in aggregate, the male–female differences in growth at birth are smaller than for survival and net migration. It is also worth noting that sex differences in net migration have the largest spread or the least consistent effect on sex ratios across these countries.

Table 2: **Sex ratios in 2010 and 2020 and decomposition of the change into contributions of male-to-female comparison of growth rate at birth and changes in the probabilities of survival and net migration. Selected HMD countries**

Country	Sex ratio			Growth rate at birth			Change in survival			Change in net migration		
	2010	2020	Change	Female	Male	Difference	Female	Male	Difference	Female	Male	Difference
Norway	99.8	101.7	1.9	0.5	0.6	0.1	2.1	3.3	1.1	6.4	7.0	0.6
Sweden	99.1	101.2	2.1	0.8	1.0	0.2	2.2	3.2	1.0	5.9	6.9	1.0
Denmark	98.3	99.0	0.7	−2.4	−2.7	−0.2	3.1	3.8	0.7	3.9	4.2	0.2
The Netherlands	98.0	98.7	0.7	−1.2	−1.6	−0.4	2.1	3.5	1.3	3.6	3.3	−0.2
Australia *	99.7	99.1	−0.6	9.1	9.2	0.1	1.7	2.6	0.9	4.6	2.9	−1.7
Switzerland	96.9	98.4	1.5	0.9	0.9	0.0	2.3	3.3	1.0	6.0	6.4	0.4
England & Wales	96.6	97.8	1.2	−0.1	0.1	0.3	2.9	3.7	0.8	3.3	3.4	0.2
United States *	96.8	97.6	0.9	3.4	3.2	−0.2	1.8	2.3	0.5	1.1	1.7	0.5
Finland	96.3	97.5	1.3	−4.1	−4.7	−0.6	3.6	4.9	1.3	3.0	3.5	0.6
Spain *	97.8	96.2	−1.6	−5.4	−5.6	−0.2	4.7	5.2	0.5	3.2	1.4	−1.8
Scotland	93.9	95.1	1.3	−6.9	−6.8	0.1	3.3	4.2	0.9	6.9	7.1	0.3
Italy	93.8	95.0	1.1	−8.3	−8.6	−0.3	4.5	5.6	1.1	3.2	3.5	0.3
France	93.9	93.8	−0.1	1.1	1.0	−0.2	3.1	4.1	1.0	−0.5	−1.4	−0.9

Source: Author's calculations applying Equation (7). Based on data from HMD (2023), CDC (2020), Bell and Miller (2005), and the US Census (2016).
Note: All values multiplied by 100. Ordered as in Table 1. * Sex ratio information available up to age 100 for Spain and the United States, and up to age 88 for Australia. Positive values in the components (differences between male and female values in growth rate at birth, survival, and net migration) correspond to elements that contribute to the increase in the sex ratio, while negative values contribute to the decline in the sex ratio.

A final disaggregation of the changes in the sex ratio is the age decomposition presented in Figure 3. Over ages a black line corresponds to the age-specific contribution to the change over time in sex ratios. The positive and negative components of the population growth rate (growth at birth – blue, change in survival – red, change in net-migration – green) combine to obtain this age-specific contribution. Several additional interesting findings should be noted: (1) the positive survival contribution, reflecting the catching-up of males' to females' lower mortality values, observed in our four selected populations and with most noticeable impacts between ages 50 and 85, although this effect is more modest in the United States than for the other populations; (2) the mortality crossover effect around ages 87–89, when for Sweden and the Netherlands the male catching up in survival before that age changes into a widening of the female survival advantage; (3) the prominent downward pressure exerted by differential net migration in Australia, and in the Netherlands with ages with greater female changes (negative) cancelling out other ages with more male prominent changes (positive) (Table 2 net-migration component of –0.2 for this country); (4) in contradistinction, greater increases in male than in female net migration in Sweden and the United States (with the same value as the survival differential between sexes in Table 2); (5) some contrasting growth at birth, highlighted by older cohorts in Sweden and the United States. This last point deserves a special note. Growth rates at birth and population composition combine in the first row in Equation (7) to return their contribution to the change in sex ratios. Although males and female growth rates at birth show only minor deviations, at older ages there are substantial population composition gaps between females and males. For example, 14% of Swedish women are between ages 70 and 90, but only 11% of males. A similar disaggregation as that performed in Equation (4) using Kitagawa decomposition shows that the composition of the population is the main explanatory factor here (sensitivity analysis for the other components did not show such a relevant role for the composition of the population; see Figure A-3 for growth at birth and Figure A-4 for survival). In the case of the United States this becomes the most important component at older ages, when survival and net migration are not present given equal accumulated cohort changes for females and males.

Figure 3: **Age-decomposition of change in sex ratio between 2010 and 2020 into contributions of growth rate at birth and changes in the probabilities of survival and net migration. Australia, Sweden, the Netherlands, and the United States**
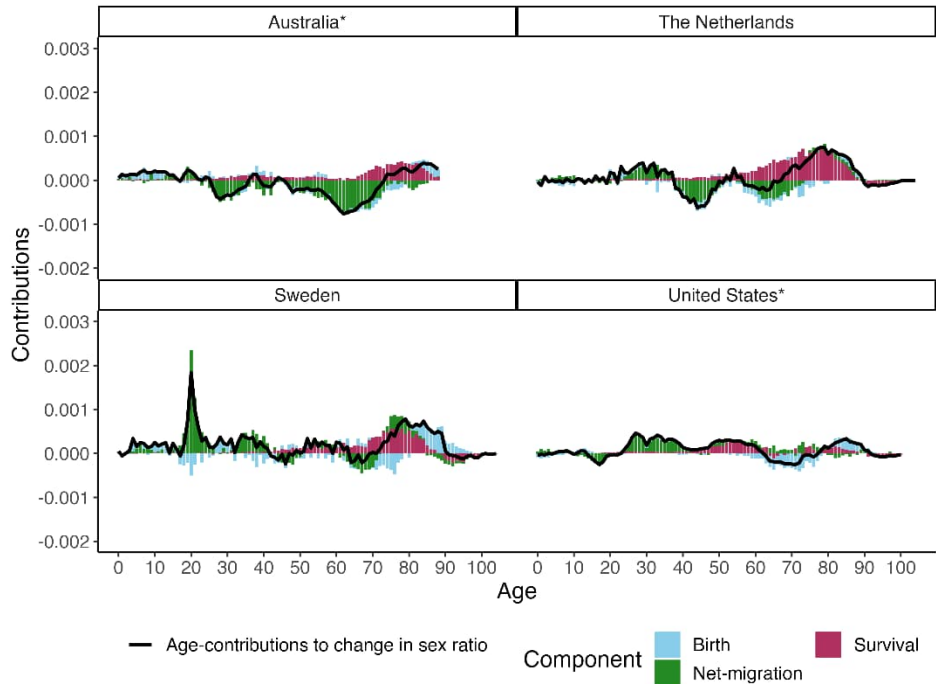
Source: Author's calculations applying Equation (7). Based on data from HMD (2023), CDC (2020), Bell and Miller (2005), and the US Census (2016).
Note: * Sex ratio information available up to age 100 for the United States, and up to age 88 for Australia. Positive values signify a greater number of males than females and negative values the opposite.

5. Discussion

The decompositions shown here illuminate how historical demographic dynamics compound over time to affect the male:female sex ratio. The sex ratio is increasing for most of these countries, primarily due to the gap closing between male and female survival rates. Indeed, some previous work has noticed the increasing sex ratio especially in Scandinavian countries and has found evidence for effects of mortality (Hollingshaus et al. 2019; Spoorenberg 2016). This narrowing of mortality between the sexes is

attributed mainly to progress in reducing heart diseases and neoplasms that result in premature deaths for males (Feraldi and Zarulli 2022). But our model extends beyond mortality, and the results show that changes in sex ratios at birth (captured in the births component) and net migration are also important.

The growth-rate-at-birth component had a minor overall effect on the change of the sex ratio (Table 2). However, when the age-specific contributions were estimated (in Figure 3) there were some pronounced contributions at older ages caused by the contrasting composition of the female and male populations at older ages. The relevant role of the population composition was also highlighted in the disaggregation of the differences in female and male growth rates (Table 1). By contrast, though sex ratios at birth may change in response to environmental conditions, they have comparatively low variability over time in many low-mortality countries (see Figure 2 and A-3), consistent with prior explorations (Hollingshaus et al. 2019). Thus, further research on the fertility, mortality, and migration histories that shaped the population compositions could contribute to the investigation of changes in sex ratios.

The migration effects are especially interesting, given the considerable variability observed across countries. This variability likely reflects migration's strong reliance on economic specialization and rapid policy changes. For the most part, migration tended to increase the overall sex ratio, only substantially decreasing it for Australia and Spain. But age-specific comparison of the four selected countries revealed even more fascinating dynamics. While Australia and the United States both have comparatively high fertility rates for high-income countries, their sex-specific migration patterns have decreased the sex ratio in Australia and increased it in the United States. This finding appears to be consistent with previous work. US migration policy post-1980 tended to bring males, especially agricultural workers, from Mexico and the rest of Latin America and the Caribbean (Gordon 2005), while Australia had more female migrants arriving from China, Vietnam, and the Philippines, and fewer of them returned to their countries of origin (Raymer et al. 2018). Sweden has had low levels of historic immigration (McAuliffe and Triandafyllidou 2022) but received a considerable number of refugee immigrants around 2015, many of them young males (Connor 2016); this abrupt change produced the migration component's 'spike' around age 20 in Figure 2.

This study constitutes only a brief demonstration of methods, and additional research should investigate the components of sex ratios in greater depth. Future research might also expand our method, which is easily adaptable, to further separate effects. Canudas-Romo, Shen, and Payne (2022) refined the variable-r method to address criticisms that the mortality component affects age structure, which indirectly affects birth rates (Lee and Zhou 2017). Since migrants often have different birth and death rates than the native-born population, the net-migration component might exert similar confounding effects. The component of changes in survival could be further separated

into averted cause-of-death (Beltrán-Sánchez, Preston, and Canudas-Romo 2008) components to see how they affect the sex ratio, though it would require considerable data. Finally, while this paper focuses on the sex ratio over all ages, our method is easily adapted to the many different age-specific sex ratios employed by researchers (Schacht et al. 2022).

Limitations of our analysis of sex ratios should be acknowledged. The initial age-specific sex ratios partition and the decomposition of the sex ratio change over time into growth rate and population composition effects (Equations 1, 3–5) only require data for one and two points in time, respectively. However, the variable-r perspective demands long cohort series of survival and net migration and their initial birth counts (Equations 2, 6–7). Problems of age-misreporting can be particularly important for historical information, and the Human Mortality Database, used in our analysis, gives warnings about these problems for each country. However, this is also the most accurate data available that consistently checks for age-misreporting and bias in the information. A possible way to elude problems of age-misreporting is to work with age groups instead of single ages, and our methodology can adapt to such situations.

6. Conclusion

This paper contributes to the body of work involving decomposition methods and variable-*r* decomposition in particular. It presents a set of new equations that can be used to assess sex imbalances in populations via the historical components of change experienced by cohorts. We implemented these new methods and found that while the components of change often affect sex ratios similarly across countries, there are also interesting differences and variation. Sex ratios have numerous applications, including those related to public policies. It is our hope that researchers will continue to explore the frontiers in mathematical demography to honor the legacy of Jim Vaupel.

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Appendix

Appendix 1: US data

For the United States, the HMD allowed us to build a truncated historical series from 1933 to 2020. To complete the US series from 1909 to 1932, we relied on birth data from the Centers for Disease Control and Prevention (CDC 2020), mortality data from the Social Security Administration (Bell and Miller 2005), and population estimates from the US Census Bureau (US Census 2016). The population and births data were complete and in the format required; however, the mortality data was not. From the Social Security Administration (Bell and Miller 2005) we obtained age-specific period and cohort data on the probability of dying within a year (q_x) from 1900, 1910, 1920, and 1930, for males and females separately, and yearly infant mortality data from 1900–1939. This data was interpolated linearly to create cohort q_x data. The year 1918, an exceptional year due to the Spanish Flu, was calculated separately and did not influence the interpolations for other years. Ratios of expected-to-actual mortality in 1918 were calculated from the cohort and period data available for that year and used to determine the rest of the required data for 1918. Population estimates for males and females were back-calculated using birth and probability-of-death data. These population estimates were then used to combine the probability-of-death data as one measure including both males and females. These probability-of-dying estimates (q_x) were converted to mortality rates (m_x). The formula given by Wilmoth et al. (2021: 37) was used to estimate a_0 for males and females, with the value of q_0 used in lieu of m_0. Death count data (D_0) was calculated from q_x and population counts. The formula given by Wilmoth et al. (2021: 37) was used to determine the combined a_0 value for males and females. Using this value, q_x values were converted to m_x using the formula given in Preston, Heuveline, and Guillot (2001: 49), rearranged to make q_x the subject.

Appendix 2: Approximating continuous change

We approximated three types of quantities: the demographic function at an exact time point (i.e., midpoint), the derivative for a demographic function, and the relative derivative of change. We implemented previously published standard approximations (Vaupel and Canudas-Romo 2002 and 2003).

We approximate the intensity, or relative derivative, assuming a constant rate of change between populations. Given a demographic function *v(x,y)* measured for two populations *y* and *y+h*, we approximate the relative derivative of the function at the midpoint with:

$$\acute{v}(x, y + h/2) \approx \frac{\ln\left[\frac{v(x,y+h)}{v(x,y)}\right]}{h}. \tag{A1}$$

This assumes a constant rate of change between populations 1 and 2. Making the same assumption, we approximate the midpoint for the function using

$$v(x, y + h/2) \approx [v(x, y)v(x, y + h)]^{1/2}, \tag{A2}$$

and the derivative by

$$\dot{v}(x, y + h/2) = \acute{v}(x, y + h/2)v(x, y + h/2). \tag{A3}$$

Figure A-1: **Decomposition of the change in the sex ratio between 2010 and 2020 into the contributions of growth rate at birth, and changes in the probabilities of survival and net-migration. Selected HMD countries**
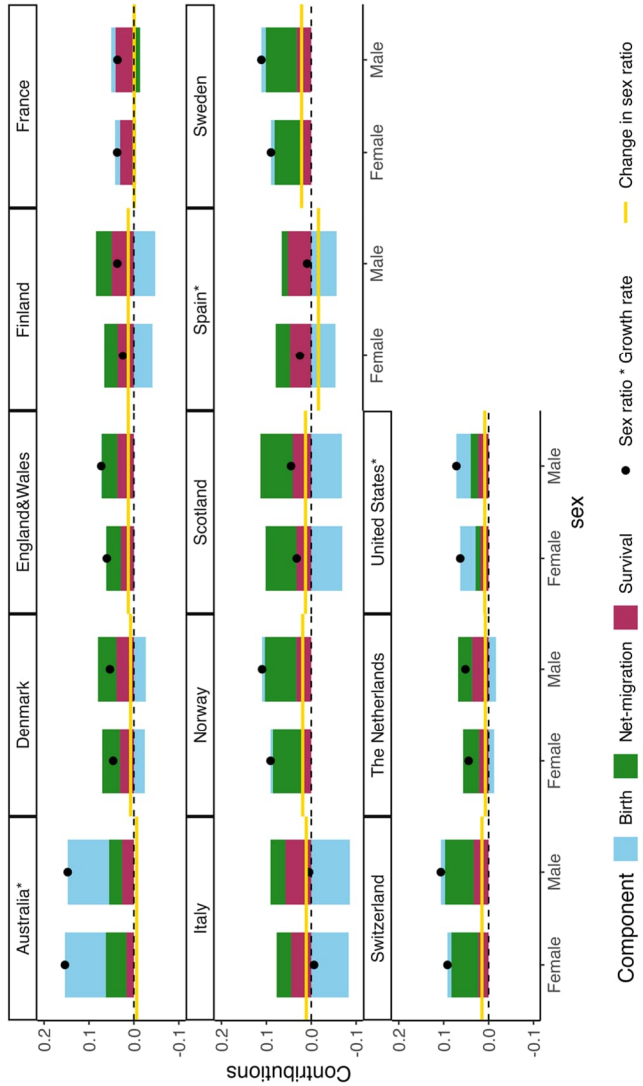
Source: Author's calculations applying Equation (7). Based on data from HMD (2023), CDC (2020), Bell and Miller (2005), and the US Census (2016).
Note: * Sex ratio information available up to age 100 for Spain and the United States, and up to age 88 for Australia. Positive values indicate greater counts of people (males or females, depending on the column) and negative values the opposite.

Figure A-2:   Age-decomposition of the change in the sex ratio between 2010 and 2020 into the contributions of growth rate at birth, and changes in the probabilities of survival and net-migration. Selected HMD countries
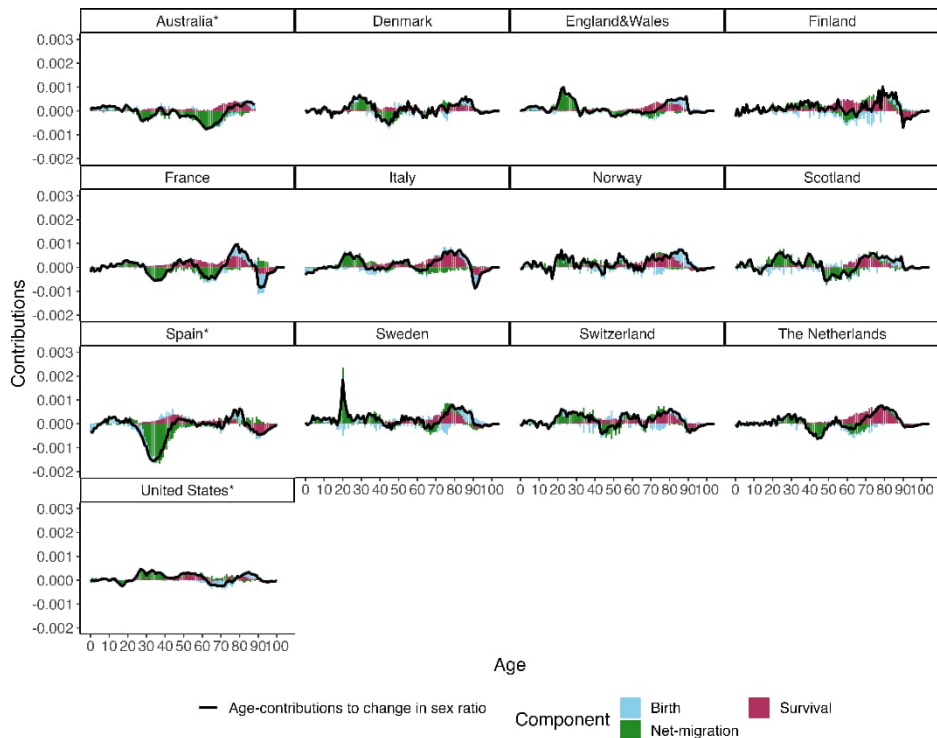
Note: * Sex ratio information available up to age 100 for Spain and the United States, and up to age 88 for Australia. Positive values indicate a greater number of males than females and negative values the opposite.

Figure A-3: Kitagawa decomposition of the age-specific contribution of the growth rate at birth ($r_{B,M}$ and $r_{B,F}$) multiplying population composition (c_M and c_F) components, which combined contribute to the change in the sex ratio (first row in Equation (7)). Sweden 2010 to 2020

Note: Positive values indicate a greater number of males than females and negative values the opposite.

Figure A-4: **Kitagawa decomposition of the age-specific contribution of survival improvements ($r_{\ell,M}$ and $r_{\ell,F}$) multiplying population composition (c_M and c_F) components, which combined contribute to the change in the sex ratio (second row in Equation (7)). Sweden 2010 to 2020**
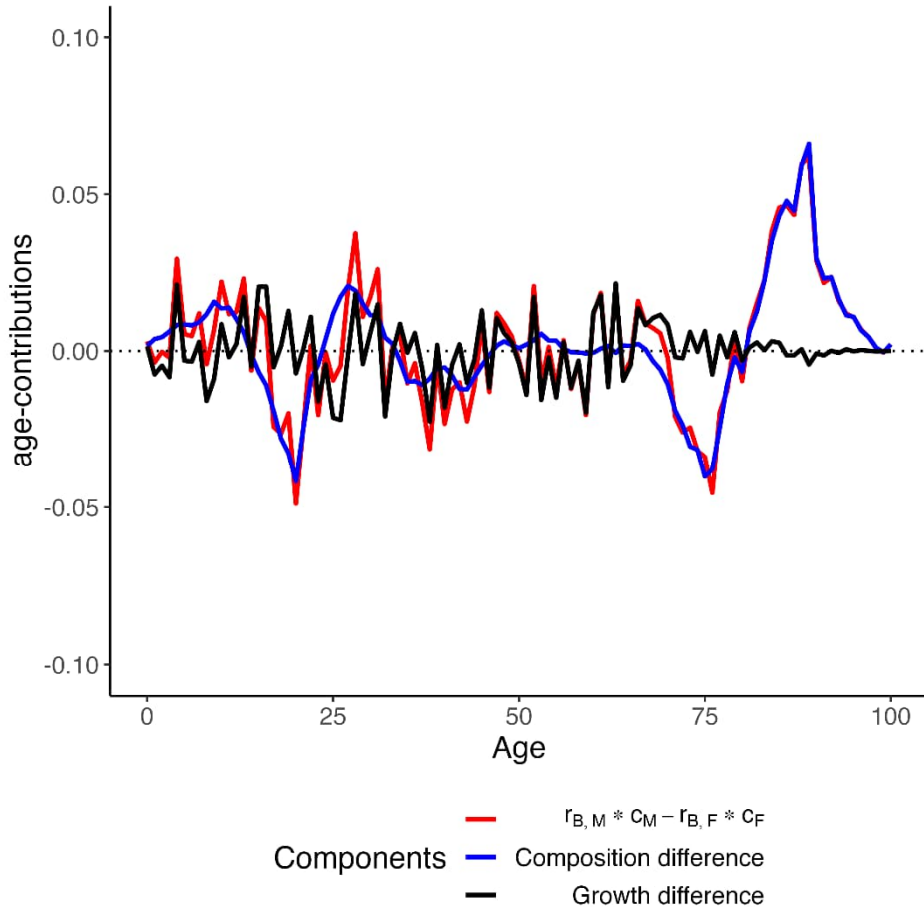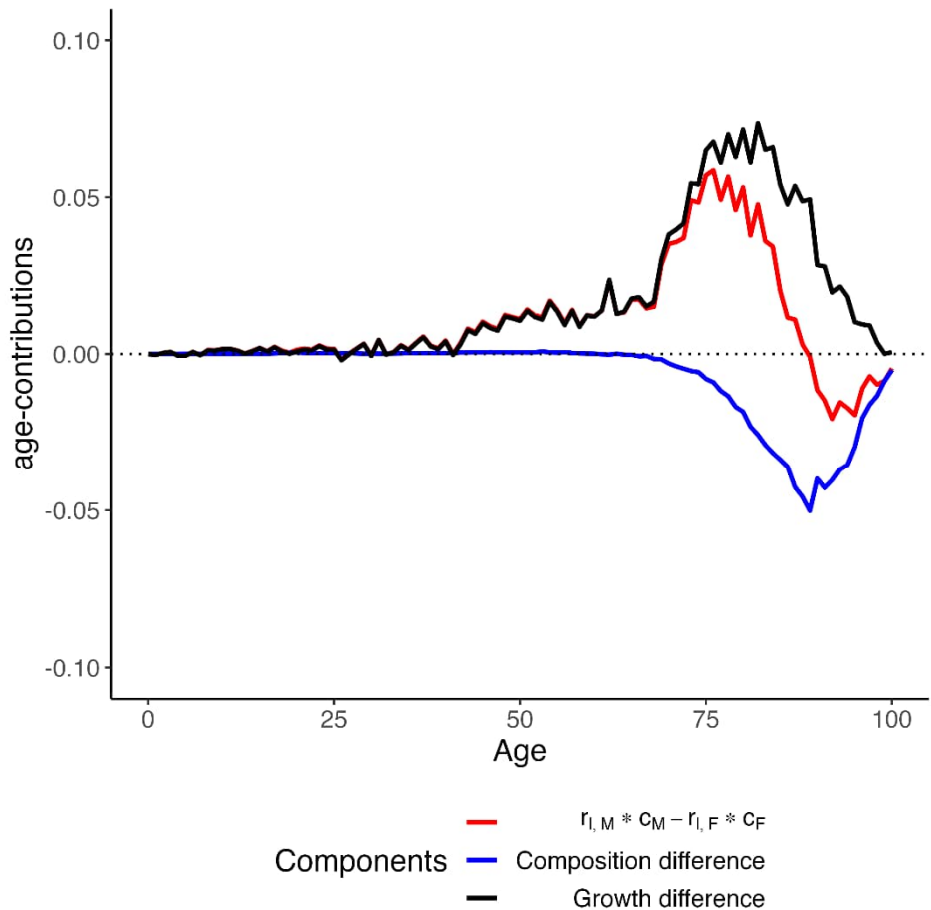
Source: Author's calculations applying second row of Equation (7) and Kitagawa decomposition similar to Equation (5). Based on data from HMD (2023).
Note: Positive values indicate a greater number of males than females and negative values the opposite.