

# DEMOGRAPHIC RESEARCH

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*Research Article*

**Demography and the statistics of lifetime  
economic transfers under individual  
stochasticity**

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# Demography and the statistics of lifetime economic transfers under individual stochasticity

Hal Caswell<sup>1</sup>

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## Abstract

### BACKGROUND

As individuals progress through the life cycle, they receive income and consume goods and services. The age schedules of labor income, consumption, and life cycle deficit reflect the economic roles played at different ages. Lifetime accumulation of economic variables has been less well studied, and our goal here is to rectify that.

### OBJECTIVE

To derive and apply a method to compute the lifetime accumulated labor income, consumption, and life cycle deficit, and to go beyond the calculation of *mean* lifetime accumulation to calculate statistics of *variability* among individuals in lifetime accumulation.

### METHODS

To quantify variation among individuals, we calculate the mean, standard deviation, coefficient of variation, and skewness of lifetime accumulated transfers, using the theory of Markov chains with rewards (Caswell 2011), applied to National Transfer Account data for Germany of 1978, and 2003.

### RESULTS

The age patterns of lifetime accumulated labor income are relatively stable over time. Both the mean and the standard deviation of remaining lifetime labor income decline with age; the coefficient of variation, measuring variation relative to the mean, increases dramatically with age. The skewness becomes large and positive at older ages. Education level affects all the statistics. About 30% of the variance in lifetime income is due to variance in age-specific income, and about 70% is contributed by the mortality schedule. Lifetime consumption is less variable (as measured by the CV) than lifetime labor income.

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## **CONCLUSIONS**

We conclude that demographic Markov chains with rewards can add a potentially valuable perspective to studies of the economic lifecycle. The variation among individuals in lifetime accumulations in our results reflects individual stochasticity, not heterogeneity among individuals. Incorporating heterogeneity remains an important problem.

## **1. Introduction**

Many countries are in the midst of an age transition, following a trajectory from an age structure dominated by the young, to an age structure with a large component in the productive ages, and finally to a structure dominated by the old. Such a transition has far-reaching effects on the sustainability of public transfer systems, public pensions and health and long-term care budgets.

The ageing of populations has naturally focused attention on the relation between demography and the generational economy. The generational economy comprises production, consumption, sharing, and saving of resources across age (Lee and Mason 2011). Production (in the form of labor income) and consumption have received a great deal of attention, and are being studied using data from National Transfer Accounts (NTA).

Labor income and consumption have generally been studied in two complementary ways. One is by comparisons of the age patterns of per capita income or consumption, jointly referred to as the economic lifecycle. The second is in terms of population aggregates, in which age-specific per capita income and consumption are weighted by the population age distribution (Skirbekk, Loichinger, and Barakat 2012) to obtain values at the population level.

The international and comparative National Transfer Account project (NTA; for more details see [www.ntaccounts.org](http://www.ntaccounts.org)) presents data on age-specific economic variables such as consumption, income, transfers, or assets. They include all possible actors, such as governments, families and firms, and are therefore able to combine the examination of public and private redistribution of resources. Using these data, it is possible to picture the economic life cycle of individuals and to study, for example, the impact of changes in the age structure on the economy or the impact of institutional settings on the individual economic lifecycle. These studies are based on the idea that an individual has some level of income, consumption, and deficit (the difference between income and consumption), and that, aggregated over a population, these generate transfers of resources among age classes. Studies of these transfers determine those periods in which labor income is insufficient to finance an individual's consumption (i.e., periods of dependency), and how those periods are changed by public and private transfers or asset-based reallocations,

including saving and investment (e.g., Lee 1994; Lee, Lee, and Mason 2006; Lee and Mason 2011).

Our goal here is to introduce two new perspectives on the generational economy. First, we develop measures for the *lifetime accumulation* of income, consumption, and deficit, as a consequence of the economic lifecycle. The accumulated values are obtained by integrating age-specific income and consumption over age, taking into account the probabilities of survival (or other transitions, if relevant). Such lifetime accumulations are familiar in demography. An individual's accumulated reproductive output is the total fertility rate (TFR) if mortality is ignored, or the net reproductive rate ( $R_0$ ) if mortality is incorporated (Caswell 2011). In our analysis here, the age schedules of income or consumption take the place of the age schedule of fertility, and we will show how to compute the lifetime accumulation of these economic measures.

Our second goal is to go beyond calculating *expectations* of lifetime accumulations (as, e.g.,  $R_0$  is the expectation of lifetime reproduction) to examine variation, among individuals, in lifetime accumulation. This variation exists for two reasons. Consider a cohort of individuals, all experiencing exactly the same demographic rates and the same patterns of income and consumption. Even though the rates are the same for all individuals, mortality is a stochastic process, and individuals will differ in how long they live.<sup>3</sup> In addition, the income, consumption, and other variables accruing to an individual at a given age are themselves random variables that can be characterized by their moments. So, even two identical individuals who live to identical ages will differ in their lifetime accumulation of economic variables. The variation in lifetime outcomes resulting from these processes is called *individual stochasticity* (Caswell 2009, 2011).

Individual stochasticity must not be confused with heterogeneity among individuals. Individual stochasticity produces variation among individuals that are all experiencing exactly the same age- or stage-dependent vital rates. Unobserved heterogeneity can act to amplify that variation, but its calculation requires models that include both observed and unobserved heterogeneity [e.g., frailty models in survival analysis (Caswell 2014a)]. Empirical measures of the variation in accumulated rewards will reflect both individual stochasticity and heterogeneity; one of the values of our approach is its potential to separate the two sources of variation (Caswell 2011).

To quantify the effects of individual stochasticity, we will calculate the variance, standard deviation, coefficient of variation, and skewness of lifetime accumulated economic variables. Such information has potential uses in policy-related analyses. Calculations based solely on mean values provide no information on the risks associated with variable outcomes. Knowing the mean lifetime income or consumption does not reveal how variable that accumulation will be among members of a cohort, and hence says nothing about how common unusually high or unusually low values will be among members of

<sup>3</sup>In a multistate model, individuals would also differ in how long they spend in each state (Caswell 2006, 2009), but we know of no multistate economic transfer data from which to develop such models.

a cohort. Skewness (the standardized third moment about the mean) provides extra information beyond variance; positive skewness implies a distribution with a long positive tail, and negative skewness implies the opposite. A convenient reference point is that the exponential distribution has a skewness of 2. The approach we will introduce provides, if desired, all the moments of remaining lifetime accumulation, so kurtosis and other functions of the higher moments could also be calculated if desired (Caswell 2011).

The results reported here, based on data from Germany, are the first exploration of accumulations over the economic lifecycle. As such, we have little to which to compare them. We expect that if more examples are analyzed, patterns will begin to appear in the comparative results.

**Organization of the paper.** In Section 2 we describe the German NTA data on which our analyses will be based. In Section 3 we describe our methods, using results from the demographic version of the theory of Markov chains with rewards. In Section 4 we present a series of applications to German National Transfer Accounts data (Kluge 2011). We close with a discussion.

**Notation.** Matrices are denoted by upper-case bold symbols (e.g.,  $\mathbf{P}$ ), vectors by lower-case bold symbols (e.g.,  $\boldsymbol{\rho}$ ). Vectors are column vectors by default. The transpose of  $\mathbf{P}$  is  $\mathbf{P}^T$ . The vector  $\mathbf{1}$  is a vector of ones. The diagonal matrix with the vector  $\mathbf{x}$  on the diagonal and zeros elsewhere is denoted  $\mathcal{D}(\mathbf{x})$ . The expected value is denoted by  $E(\cdot)$ , the variance by  $V(\cdot)$ , the coefficient of variation by  $CV(\cdot)$  and the skewness by  $Sk(\cdot)$ . The Hadamard, or element-by-element, product of matrices  $\mathbf{A}$  and  $\mathbf{B}$  is denoted by  $\mathbf{A} \circ \mathbf{B}$ . Transition matrices of Markov chains are written in column-to-row orientation, and hence are column-stochastic.

## 2. Data

Our analyses are based on NTA estimates for Germany obtained from the German Income and Expenditure Surveys (*Einkommens und Verbrauchsstichprobe*, or EVS) of 1978 and 2003. The EVS has been conducted by the Federal Statistical Office since 1978 at five year intervals, and is based on a representative quota sample of Germany's private households. In 1978, the dataset included 46,941 households from the former Federal Republic of Germany alone. The 2003 wave includes around 50,000 households made up of some 127,000 individuals; the scientific-use file is a 98 percent sample of the original data set.

The EVS includes a detailed account of income, consumption, savings, and assets. For three months, participating households keep a detailed book of household accounts that covers every kind of potential income and expenditures. The survey is representative of households with a monthly net income of less than 18,000 euros. Very wealthy

households, persons with no permanent residence, and individuals living in institutions are not included. The number of oldest old individuals above age 85 is low, so we use the estimates only up until age 85.

From the EVS survey data we obtained the first three moments of age-specific labor income, age-specific expenditure, and age-specific deficits, for ages 0–90. We combined these with period mortality schedules (both sexes combined) from the Human Mortality Database (2012) to formulate absorbing Markov chains with rewards.

### 3. Markov chains with rewards

We analyze lifetime accumulations using the approach introduced by Caswell (2011) in a study of lifetime reproductive output. This is based on the mathematical framework of Markov chains with rewards (MCWR), introduced by Howard (1960) in the context of dynamic programming (see also, e.g., Benito 1982; Puterman 1994; Sladký and van Dijk 2005). An individual moves among states according to a finite-state Markov chain. In our case, the states consist of age classes, plus an absorbing state representing death. The probability of transition from age class  $i$  to age class  $i + 1$  is the survival probability  $p_i$ , and the probability of transition from age class  $i$  to death is  $q_i = 1 - p_i$ . This age-classified structure leads to a particularly simple Markov chain, but the theory is equally applicable to more complicated stage-classified or multi-state models (Caswell 2011).

At each time, the individual collects a “reward”  $r_{ij}$  that depends on the transition, from state  $j$  to state  $i$ , realized at that time. The reward may be positive or negative; in our case, the rewards will represent income, consumption, or the deficit (the difference between income and consumption). We assume that no rewards accrue to individuals who have reached the absorbing state.

The transition matrix of the absorbing Markov chain is, in general,

$$\mathbf{P} = \left( \begin{array}{c|c} \mathbf{U} & \mathbf{0} \\ \mathbf{M} & \mathbf{I} \end{array} \right), \quad (1)$$

where  $\mathbf{U}$  is the transition matrix (dimension  $s \times s$ ) among transient states and  $\mathbf{M}$  a matrix of mortality rates, with  $m_{ij}$  the probability of death from cause  $i$  in age class  $j$ . In our case,  $\mathbf{U}$  has survival probabilities on the subdiagonal and zeros elsewhere,

$$\mathbf{U} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ p_1 & 0 & \cdots & 0 \\ & \ddots & & \\ 0 & \cdots & p_s & 0 \end{pmatrix} \quad (2)$$

and  $\mathbf{M}$  is a row vector of mortalities given by

$$\mathbf{M} = \mathbf{1}^\top - \mathbf{1}^\top \mathbf{U}. \tag{3}$$

Provided that the dominant eigenvalue of  $\mathbf{U}$  is less than 1, which we assume and which is true for demographic applications, an individual beginning in any transient state will eventually be absorbed (i.e., will eventually die) with probability 1.

The rewards collected at a transition are, in general, random variables. Let the random reward obtained by an individual moving from state  $j$  to state  $i$  be  $r_{ij}$ . Let the matrix<sup>4</sup> of the  $k$ th moments of the  $r_{ij}$  be denoted  $\mathbf{R}_k$ :

$$\mathbf{R}_k = \left( E \left[ r_{ij}^k \right] \right). \tag{4}$$

Define  $\boldsymbol{\rho}$  as a vector whose entries are the accumulated rewards accruing to an individual starting in each state of the Markov chain, and let  $\boldsymbol{\rho}_k$  be the vector of the  $k$ th moments of the entries of  $\boldsymbol{\rho}$ ,

$$\boldsymbol{\rho}_k = \left( E \left[ \rho_i^k \right] \right). \tag{5}$$

The calculation of the accumulated rewards proceeds in the “backwards” fashion familiar from dynamic programming (Howard 1960). Choose some terminal time  $T$ , define  $t$  as the time remaining until this terminal time, and let  $\boldsymbol{\rho}(t)$  be the reward yet to be accumulated at  $t$ . At the terminal time, no more rewards will be accumulated, so  $\boldsymbol{\rho}(0) = 0$ .

Caswell (2011) showed that the first three moments of the accumulated reward satisfy the following system of equations:

$$\boldsymbol{\rho}_1(t+1) = (\mathbf{P} \circ \mathbf{R}_1)^\top \mathbf{1} + \mathbf{P}^\top \boldsymbol{\rho}_1(t) \tag{6}$$

$$\boldsymbol{\rho}_2(t+1) = (\mathbf{P} \circ \mathbf{R}_2)^\top \mathbf{1} + 2(\mathbf{P} \circ \mathbf{R}_1)^\top \boldsymbol{\rho}_1(t) + \mathbf{P}^\top \boldsymbol{\rho}_2(t) \tag{7}$$

$$\boldsymbol{\rho}_3(t+1) = (\mathbf{P} \circ \mathbf{R}_3)^\top \mathbf{1} + 3(\mathbf{P} \circ \mathbf{R}_2)^\top \boldsymbol{\rho}_1(t) + 3(\mathbf{P} \circ \mathbf{R}_1)^\top \boldsymbol{\rho}_2(t) + \mathbf{P}^\top \boldsymbol{\rho}_3(t) \tag{8}$$

for  $t = 0, \dots, T-1$ , with  $\boldsymbol{\rho}_1(0) = \boldsymbol{\rho}_2(0) = \boldsymbol{\rho}_3(0) = 0$ . The moments of the lifetime accumulated reward are obtained as the  $\lim_{t \rightarrow \infty} \boldsymbol{\rho}_i(t)$ . In practice, the system of equations (6)–(8) is iterated until the values converge, to obtain the vectors of moments of lifetime accumulation.

From these moment vectors we calculate some descriptive statistics of the lifetime accumulation, including the mean, standard deviation, coefficient of variation (CV) and skewness.

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<sup>4</sup>In our notation, the expression  $(x_{ij})$  denotes a matrix with  $x_{ij}$  in the  $i$ th row and  $j$ th column.

$$\text{mean: } \rho_1 \quad (9)$$

$$\text{variance } V(\rho): \rho_2 - (\rho_1 \circ \rho_1) \quad (10)$$

$$\text{standard deviation } SD(\rho): \sqrt{\rho_2 - (\rho_1 \circ \rho_1)} \quad (11)$$

$$\text{coefficient of variation } CV(\rho): SD(\rho) \mathcal{D}(\rho_1)^{-1} \quad (12)$$

$$\text{skewness } Sk(\rho): \mathcal{D}\left(V(\rho)\right)^{-\frac{3}{2}} [\rho_3 - 3\rho_1 \circ \rho_2 + 2\rho_1 \circ \rho_1 \circ \rho_1]. \quad (13)$$

The standard deviation and the coefficient of variation quantify variability in lifetime accumulation, on an absolute and a relative scale respectively. Skewness is less commonly used; it measures the asymmetry of the distribution. The skewness of a symmetrical distribution is zero; positive skewness implies a longer tail of values to the right, and negative skewness a longer tail of values to the left. A helpful point of reference is that the exponential distribution has a skewness of 2.

## 4. Economic transfers for Germany

We apply the MCWR methodology to data on schedules of age-specific income, consumption, and the corresponding deficit by age based on National Transfer Accounts for 1978 and 2003 for Germany. We obtained the first three moments of the age patterns from the Income and Expenditure Survey of the respective year. We constructed the Markov chain matrices  $\mathbf{U}$  and the  $\mathbf{M}$  from age-specific mortality schedules  $q_i = 1 - p_i$  obtained from the Human Mortality Database (Human Mortality Database 2012).

Our primary focus is on labor income, and we begin by examining changes in lifetime income between 1978 and 2003 (Section 4.1), and differences due to educational level (Section 4.3). We decompose the variance in lifetime income into components according to (1) variation in age-specific income over the economic lifecycle and (2) variation in the length of life (Section 4.2). We also examine the effects of changes in the mortality schedule (Section 4.4), and conclude by comparing lifetime accumulations of income, consumption, and the lifecycle deficit (Section 4.5).

### 4.1 Changes in economic transfers over time

Figure 1 shows the mean, standard deviation, coefficient of variation, and skewness of per capita age-specific labor income in 1978 and 2003. Only minor differences are apparent between years; working ages remain relatively unaffected and stable. Individuals started to work a little earlier on average in 1978, but the exit slopes of mean income are quite

similar. It is not surprising that entry ages do not differ substantially, as children are not able to work and only slight changes occur due to a higher fraction of young individuals proceeding to tertiary education. Still, labor market exit ages are predominantly determined by welfare policies. Life expectancy at birth increased by six years within the 25 years of observation, and no adjustment has taken place at retirement age; exit ages are not even more volatile. Mean income reaches a peak much later in 2003 than in 1978, but in both cases prime working ages are around ages 45–50. The standard deviation follows a similar pattern, but with higher fluctuation at the beginning and end of the working life in 1978 as compared to 2003. When inter-individual variation is measured relative to the mean using the CV, it is greater at early and late ages than at ages during the middle of working life.

Figure 2 shows the statistics of (remaining) lifetime income as a function of age. The mean lifetime income at birth was approximately  $7 \times 10^5$  € in 1978, and had increased to approximately  $8 \times 10^5$  € by 2003. Mean remaining lifetime income begins to decline at about age 25, and approaches 0 at about age 65.

The standard deviation of income also declines with age. The increase in mean lifetime income at birth from 1978 to 2003 was accompanied by a small reduction in the standard deviation (from approximately  $2 \times 10^5$  to  $1.7 \times 10^5$ ). The CV, which scales the standard deviation relative to the mean, remains low (less than 1) until approximately age 60, at which point it climbs sharply, reaching values as high as 10 by age 85. The skewness of remaining lifetime income also increases sharply after age 60, to large positive values. That is, remaining lifetime income at older ages is very variable, relative to its mean, and its distribution is very positively skewed. The distribution of income and the schedule of mortality combine to produce, on the basis of stochasticity alone, a few rare, fortunate individuals with unusually large remaining lifetime incomes.

Table 1 compiles these figures for ages 0 (birth), 18 (about the start of working life), 45 (peak of working life), and 65 (end of working life). By age 45, mean lifetime income has declined by about half (55%), and the standard deviation has declined by about 40%. The biggest change in the statistics of lifetime income is apparent by age 65, by which point the CV has increased by an order of magnitude and the skewness has gone from  $-0.1$  (a very symmetrical distribution) to a high positive skewness of 4.9. At age 65, the future income accumulation of a few individuals will be very large relative to the mean.

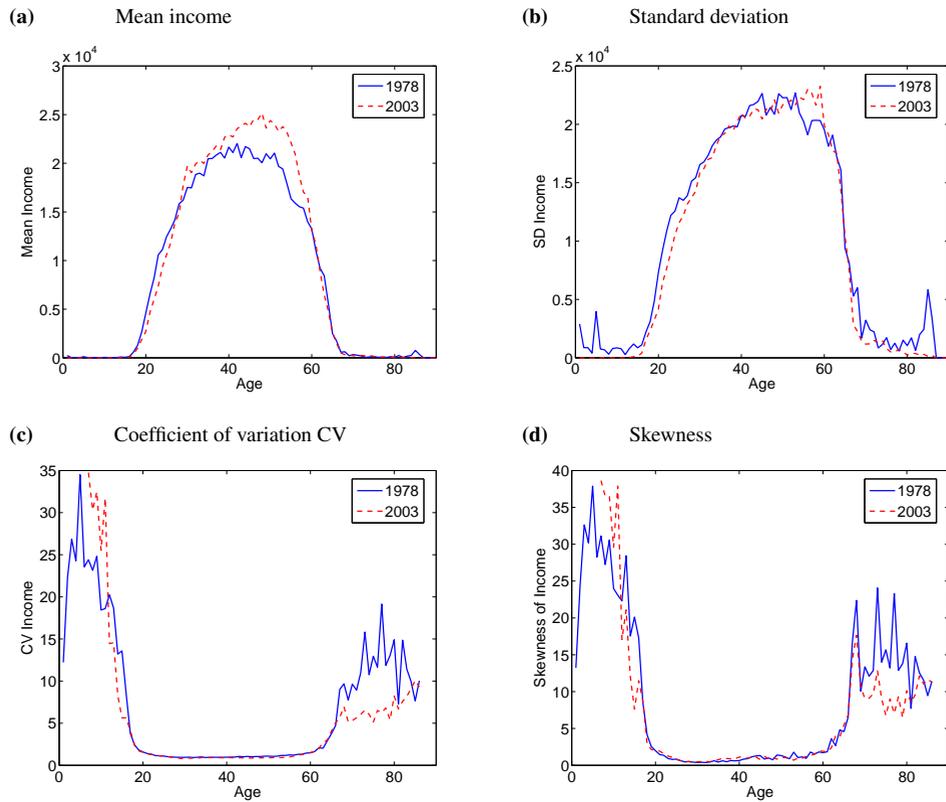
**Figure 1:** Age schedules of the statistics of per capita income in Germany, in 1978 and 2003

(a) Mean age-specific income (€)

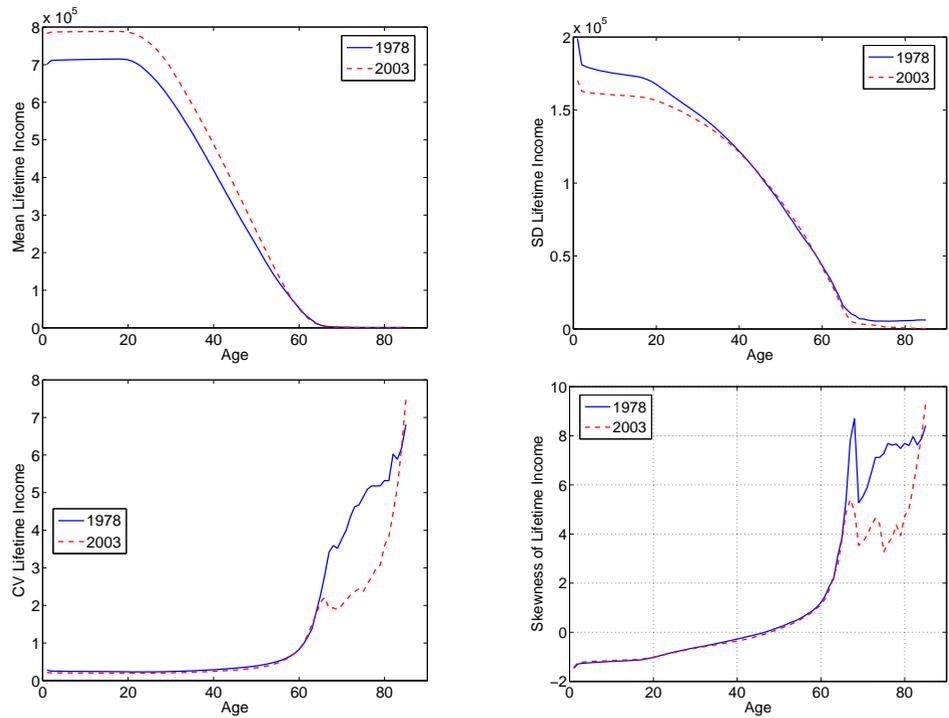
(b) The standard deviation of income (€)

(c) The coefficient of variation (CV) of income (dimensionless)

(d) The skewness of income (dimensionless).



**Figure 2:** Statistics of remaining lifetime accumulated income, as a function of age, in Germany in 1978 and 2003  
(a) Mean lifetime income (€)  
(b) Standard deviation of lifetime income (€)  
(c) Coefficient of variation of lifetime income (dimensionless)  
(d) Skewness of lifetime income (dimensionless).



**Table 1:** The statistics of remaining lifetime income, as a function of age, for Germany in 1978 and 2003. Mean and standard deviation measured in units of  $10^5$  €. The coefficients of variation (CV) and skewness are dimensionless.

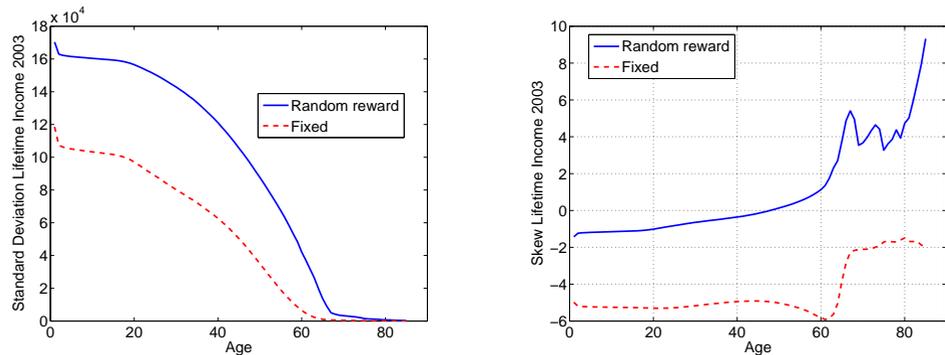
Income 1978				
Age	Mean	SD	CV	Skew
0	7.009	1.990	0.284	-1.466
18	7.146	1.694	0.237	-1.065
45	2.962	1.012	0.342	-0.032
65	0.047	0.131	2.801	5.390
Income 2003				
Age	Mean	SD	CV	Skew
0	7.834	1.704	0.218	-1.430
18	7.876	1.574	0.200	-1.048
45	3.516	1.024	0.291	-0.105
65	0.041	0.090	2.216	4.871

#### 4.2 Sources of variance: fixed and random rewards

The variance among individuals in lifetime rewards has two sources. One is variation in the path taken through the life cycle (in this case, from initial age until death; in a multistate model, pathways could be more complex). The second is variation in the rewards collected at each age. These two components can be partitioned by comparing the results in Figure 2, which contain both components, with results obtained by fixing the age-specific rewards at their mean values (i.e.,  $\mathbf{R}_2 = \mathbf{R}_1 \circ \mathbf{R}_1$  and  $\mathbf{R}_3 = \mathbf{R}_1 \circ \mathbf{R}_1 \circ \mathbf{R}_1$ ). When the rewards are fixed in this way, all variance is due to variation among pathways.

Figure 3 compares the standard deviation and skewness for fixed and random rewards, using the 2003 data. The standard deviation in lifetime income at birth is about  $1.18 \times 10^5$  € under the fixed reward model and about  $1.71 \times 10^5$  € under the random reward model. In other words,  $\sim 30\%$  of the overall variance is due to the randomness of the reward pattern at each age, and about  $\sim 70\%$  is due to variation in the fate (i.e., age at death of the individuals).

**Figure 3: Standard deviation and skewness of remaining lifetime accumulated income, as a function of age, for Germany in 2003, calculated under the fixed reward model and the random reward model.**



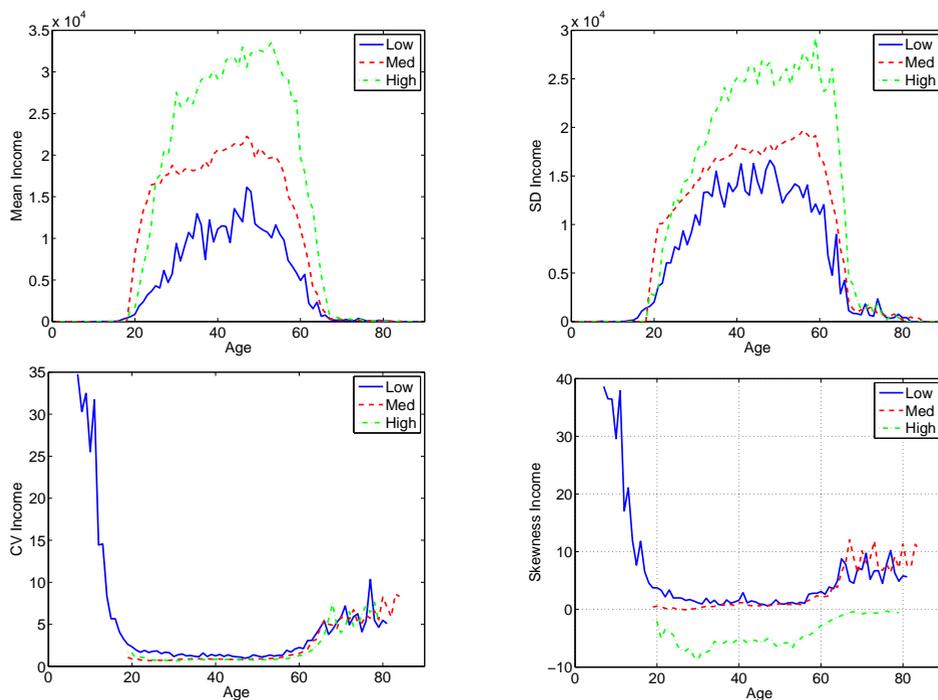
The difference in skewness is particularly noticeable. Including the variation in age-specific income schedule dramatically increases the skewness in remaining lifetime labor income.

### 4.3 The effects of education on lifetime economic transfers

Education is known to affect levels of income (Miller 1960; Becker and Chiswick 1966; Hause 1975) as does occupation (Wilkinson 1966). Here we examine effects of education on the statistics of lifetime accumulated income. Lifetime earnings are known to play an important role in intergenerational mobility (Dunn 2007). We calculated lifetime labor income using data from the EVS 2003 survey, classifying individuals into high, medium and low educational attainment categories. We grouped individuals without a completed degree in the low education category. All individuals having attained university or *Fachhochschule* fall in the high education category. The remaining individuals are grouped into medium education.

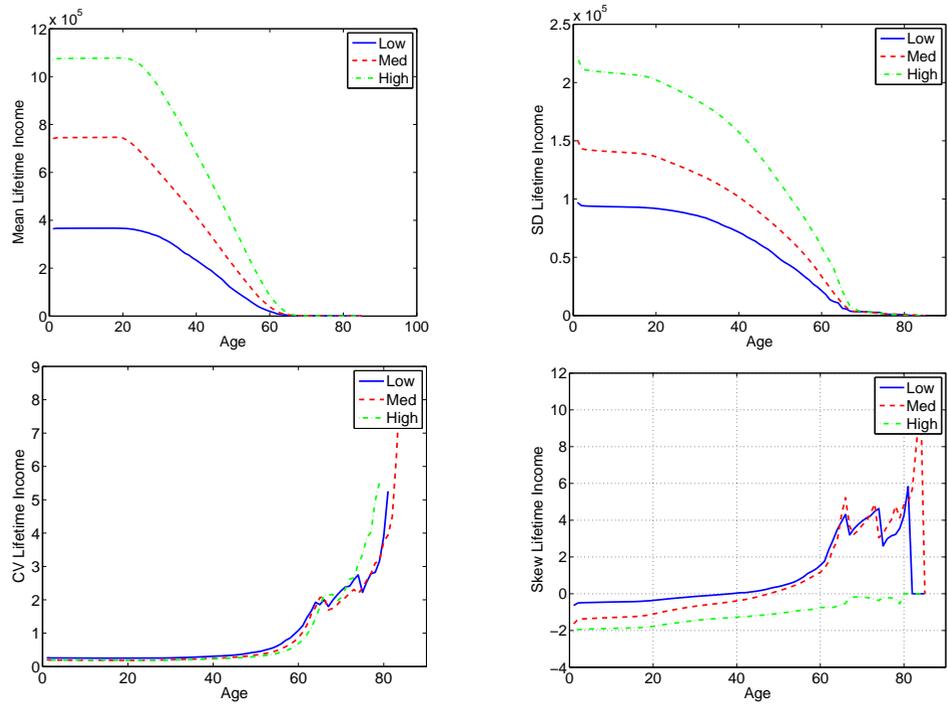
Figure 4 shows the mean, standard deviation, CV, and skewness of age-specific labor income for the low, medium, and high education level categories. As expected, mean age-specific income at any age increases with increasing educational level. The standard deviation follows the same pattern, so the CV is very similar for all three groups. The skewness for the low and medium education groups follows the pattern familiar from Figure 1. Skewness of age-specific income for the high education group is negative over much of working life, and is close to zero at older ages.

**Figure 4:** The age schedules of per capita age-specific income for low, medium, and high educational levels, from age 18 onward. Mean and standard deviation in €; CV and skewness are dimensionless.



The results for lifetime accumulated income are shown in Figure 5. The patterns of mean lifetime income are similar across all three education groups. The standard deviation declines with age, so the CV differs little among education groups, and follows a pattern similar to that shown in Figure 2. As shown in Table 2, at age 18, mean lifetime income is roughly doubled, going from low to medium education, and increased by another 40% going from medium to high education levels. The proportional differences at age 45 are similar (about 70% increase from low to medium, and again from medium to high). The benefit of high education levels for mean lifetime income are even greater at age 65 (almost doubled compared to medium education).

**Figure 5: Statistics of lifetime accumulation of income for low, medium, and high educational levels, from age 18 onward. Mean and standard deviation in €; CV and skewness are dimensionless.**



**Table 2: Statistics of remaining lifetime income as a function of age for low, medium, and high education levels. Results shown only for age 18 and older. Mean and standard deviation measured in units of  $10^5$  €. The coefficient of variation (CV) and skewness are dimensionless. Data for Germany in 2003.**

Low education				
Age	Mean	SD	CV	Skew
0	–	–	–	–
18	3.667	0.922	0.252	–0.386
45	1.654	0.590	0.357	0.189
65	0.028	0.056	2.003	4.312
Medium education				
Age	Mean	SD	CV	Skew
0	–	–	–	–
18	7.470	1.373	0.184	–1.154
45	2.943	0.854	0.290	–0.108
65	0.033	0.070	2.092	5.229
High education				
Age	Mean	SD	CV	Skew
0	–	–	–	–
18	10.782	2.037	0.189	–1.802
45	5.018	1.328	0.265	–1.166
65	0.064	0.138	2.146	–0.527

The standard deviation at each age increases with education level (Table 2), but the coefficients of variation are similar. Within each education level, skewness increases with age (becomes more positive). By age 65, the skewness of remaining lifetime income is very positive for low and medium education levels, but still slightly negative at high education levels. That is, among the elderly with high education levels, differentials in lifetime income due to good fortune will be less extreme.

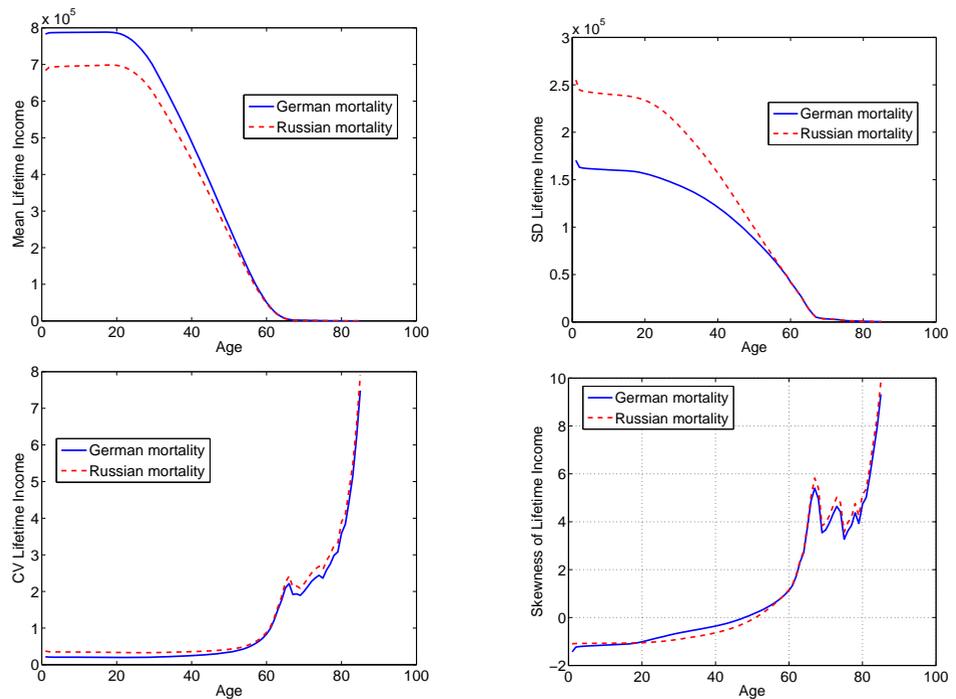
#### 4.4 Effects of changes in the mortality schedule

The statistics of lifetime income depend on both the age patterns of per-capita labor income and of mortality. By manipulating one or the other schedule, we can determine their effects. As an example, we note that in 2003, Russia had the lowest life expectancy (64.9

years) of any country in the Human Mortality Database; more than 15 years less than that of Germany. To see the effects that such mortality differences might have, we fixed the age pattern of income at the values for EVS 2003, and substituted the Russian mortality schedule for that of Germany.

The results are shown in Figure 6 and Table 3. The higher mortality schedule of Russia reduces mean lifetime income by about 12% and the standard deviation by about 35%. The differences in the CV and skewness of lifetime income are minor. This may reflect the fact that individuals usually stop working by age 65, whereas the differences in life expectancy between the two countries are due to differences in mortality outside the working years of life.

**Figure 6: Statistics of remaining lifetime accumulated income, as a function of age, using the reward schedule for Germany in 2003 and the mortality schedules of Germany and of Russia in 2003. Mean and standard deviation in €; CV and skewness are dimensionless.**



**Table 3: Statistics of remaining lifetime income as a function of age, for a hypothetical scenario combining the income data for Germany in 2003 with the mortality levels of Russia in that year. Mean and standard deviation measured in units of  $10^5$  €. The coefficient of variation (CV) and skewness are dimensionless.**

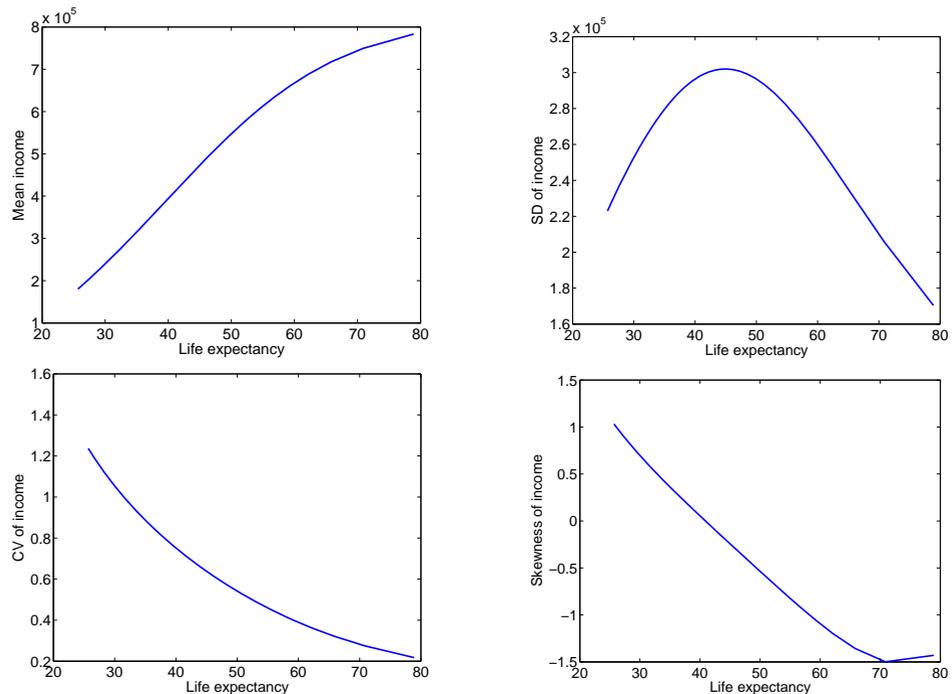
Mortality: Germany				
	Mean	SD	CV	Skew
0	7.834	1.704	0.218	-1.430
18	7.876	1.574	0.200	-1.048
45	3.516	1.024	0.291	-0.105
65	0.041	0.090	2.216	4.871

Mortality: Russia				
	Mean	SD	CV	Skew
0	6.839	2.550	0.373	-1.095
18	6.985	2.350	0.336	-1.056
45	3.196	1.229	0.385	-0.361
65	0.036	0.088	2.420	5.056

As a further exploration of the effects of mortality, we manipulated the 2003 mortality schedule of Germany by imposing an additional proportional hazard, thus maintaining the shape of the mortality schedule, while reducing life expectancy. Figure 7 shows lifetime accumulated income as a function of the life expectancy at birth of the modified mortality schedule. As expected, mean lifetime income increases with life expectancy. The standard deviation reaches a peak at intermediate mortality levels and declines at very low or very high life expectancies. The CV and skewness of lifetime income both decline as mortality declines, but the changes are modest (note the  $y$ -axis scales).

**Figure 7: Statistics of lifetime accumulated income from birth, as a function of the life expectancy resulting from a proportional mortality hazard imposed on the mortality schedule for Germany 2003. Mean and standard deviation in €; CV and skewness are dimensionless. Note the different scales on each graph.**

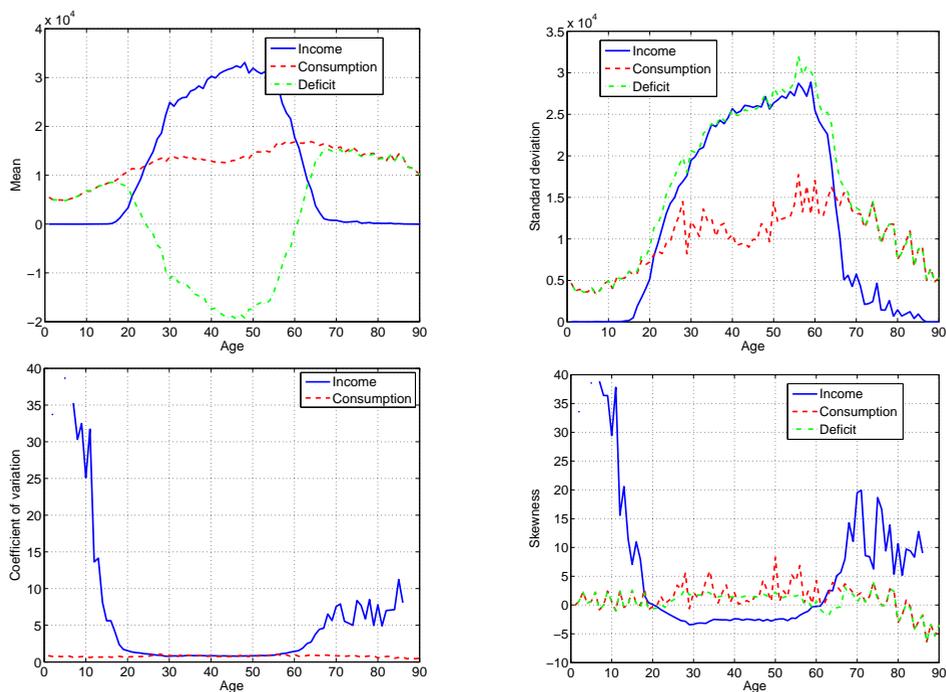


#### 4.5 Completing the economic lifecycle: income, consumption, and deficit

Labor income is, of course, only part of the picture of the economic lifecycle. We also explore the lifetime accumulation of consumption and deficit (consumption-income) for Germany using the 2003 data. This analysis differs from that in Section 4.1 because, while that analysis used only employment income, the present calculation is based on total income, including that from self-employment. The age schedules of per-capita consumption and per-capita deficit are shown in Figure 8. Income exceeds consumption

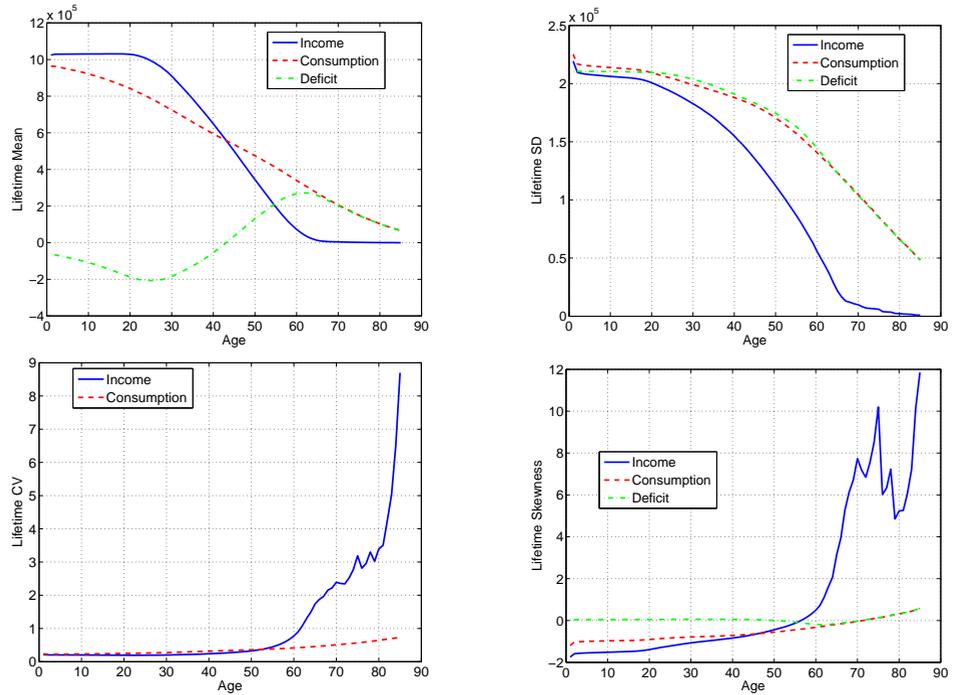
during the peak working years, but at earlier and later years, the income is less than consumption and the deficit is positive.

**Figure 8: Statistics of the age schedules of production, consumption, and deficit for Germany in 2003. Mean and standard deviation in €; CV and skewness are dimensionless. Coefficient of variation is not shown for deficit because the CV is defined only for non-negative quantities.**



The statistics of the remaining lifetime accumulation of these, as a function of age, are shown in Figure 9 and summarized in Table 4. At birth, expected lifetime income slightly exceeds expected lifetime consumption. The expectations of remaining lifetime income and of consumption both decline with age, producing a negative, and then (after age 45) positive expected lifetime deficit. The standard deviations of all three quantities decline with age.

**Figure 9:** Statistics of remaining lifetime accumulation of production, consumption, and deficit for Germany in 2003. Mean and standard deviation in €; CV and skewness are dimensionless.



**Table 4:** Statistics of remaining lifetime income, consumption, and deficit, as a function of age, for Germany in 2003. Mean and standard deviation measured in units of  $10^5$  €. Coefficient of variation (CV) and skewness are dimensionless; CV of deficit is not shown, because it is defined only for non-negative quantities.

Income				
	Mean	SD	CV	Skew
0	10.247	2.194	0.214	-1.747
18	10.302	2.021	0.196	-1.421
45	4.687	1.308	0.279	-0.634
65	0.088	0.165	1.875	3.927

Consumption				
	Mean	SD	CV	Skew
0	9.642	2.252	0.234	-1.192
18	8.514	2.103	0.247	-0.924
45	5.237	1.791	0.342	-0.629
65	2.567	1.196	0.466	-0.156

Deficit				
	Mean	SD	CV	Skew
0	-0.605	2.101	-	0.038
18	-1.788	2.097	-	0.047
45	0.549	1.816	-	0.040
65	2.478	1.189	-	-0.132

## 5. Summary

Our results are new insights from a new approach. Here, we briefly list our major findings.

1. Markov chains with rewards can be used to extend the demographic calculation of the statistics of lifetime reproduction to exactly parallel statistics of lifetime income, consumption, and deficit.
2. The age patterns of the mean, standard deviation, coefficient of variation, and skewness of lifetime are consistent across different times, differences in educational levels, and changes in mortality. This suggests that these patterns are a consistent result of the schedules of age-specific income and of age-specific mortality.

3. About 30% of the variance in lifetime income is due to variation in age-specific income, and about 70% is due to variation in the fate of individuals.
4. Changes in the skewness of remaining lifetime income are dramatic: skewness is small and negative over much of life, but by age 65 it becomes large and positive, on the order of 4–6. This is even more skewed than an exponential distribution, which has a constant skewness of 2. The only exception we found is for the high education group, where skewness at age 65 is about  $-0.5$ . Otherwise, demographic forces produce lifetime income for 50-year olds characterized by a long tail of positive deviations.

## 6. Discussion

The connection between the age-specific schedules and lifetime accumulations has proven valuable in several disparate areas of demography. When age-specific survivorship is integrated forward the result is life expectancy. When age-specific fertility is integrated forward, the result is the net reproductive rate  $R_0$  or the total fertility rate TFR, depending on whether or not mortality is included. When the age schedule of disability prevalence is integrated forward, the result is healthy life expectancy.

We can now add to this list the calculation of lifetime accumulations of economic variables. The analysis is not limited to income, consumption, and deficit; any variables for which age-specific schedules are available can be analyzed with this model. The extension from expected values to measures of variance and skewness may add an extra dimension to policy decisions, because only by accounting for variability can the risk of policy decisions be incorporated.

The results of our examples provide suggestive insights into the importance of randomness for economic events. The results summarized in Tables 1–4 reveal some patterns.

1. Labor income. The mean declines almost monotonically from birth to the end of life (not surprising). The standard deviation also declines, but more slowly, so that the CV increases dramatically with age, by about an order of magnitude between ages 45 and 65. The skewness is slightly negative until older ages; by 65 it has become large and positive. This holds true even if mortality is increased, either by using the Russian mortality schedule with its low life expectancy, or by imposing an age-independent proportional hazard. Lifetime income appears to behave differently at high levels of education, in which the skewness remains slightly negative even at age 65.
2. Consumption. The patterns for consumption differ from those for income. The mean and standard deviation both decrease with age, but not as much as is true for

income. The CV remains relatively low. Skewness remains negative and small. Thus the distribution of consumption appears to be much more symmetrical at old ages than is that of income.

3. Deficit. The mean of the remaining lifetime deficit increases with age. It is negative (i.e., lifetime income will exceed consumption) from birth to age 18, and becomes positive as an individual approaches the end of working life. The standard deviation of lifetime deficit decreases slightly, and the skewness, never very great, becomes slightly negative at the end of life.

The calculation of these lifetime statistics, following equations (6)–(8), requires the following data.

1. An age-specific schedule of mortality or, in the stage-classified case, a transition matrix among stages, including the stage-specific mortality rate. Together, these data specify the Markov chain transition matrix  $\mathbf{P}$  in equation (1).
2. The moments of the reward associated with each stage transition, expressed as the reward matrices  $\mathbf{R}_k$  in equation (4). Measuring these moments requires individual-level data. In the absence of such data, the moments can be modelled by assuming an appropriate distribution; cf., calculations of lifetime reproductive output using a Poisson assumption; (Caswell 2011).
3. We note also that lifetime accumulation is sometimes discounted as an estimate of the net present value of a specific variable. This is done, for example, to calculate the net present value of public transfers by cohorts (Bommier et al. 2010). Although we did not discount values in this analysis, it could easily be incorporated. Equations (66)–(69) in Caswell (2011) incorporate a discount factor  $0 < \beta < 1$ ; these equations would replace equations (6)–(8) above for calculations of discounted rewards.

Our results are the first to be calculated for any country. We hope that further analyses will permit comparative studies that will help to understand the patterns.

**The role of individual stochasticity.** The variance in lifetime accumulation exists even though every individual experiences the same age-specific schedules of mortality and rewards. Therefore, no heterogeneity appears in the calculations; instead the variances we report here are due strictly to the individual stochasticity implied by the demographic parameters and the distributions of age-specific rewards. These results provide a baseline against which observed variation can be compared, to quantify the effects of heterogeneity that no doubt operates in real populations.

Note that age is the only *i*-state variable (Metz and Diekmann 1986; Caswell 2001, Sec. 3.1) to appear in these calculations. Therefore, the rewards accumulated at one

age are independent of the rewards accumulated at the previous age. If they were not, individuals would experience different conditions depending on some property other than age. It is as if an individual may, conditional on survival, randomly sample the pseudo-lives of different individuals: at age  $x$  collecting the income of a wealthy banker, at age  $x + 1$  the income of an unemployed factory worker, and so on.<sup>5</sup>

This may seem strange, because individuals do not really do this, but it is an unavoidable assumption of a solely age-dependent model. Demography is replete with such calculations. Calculations of TFR or  $R_0$  assume that a woman at age  $x$  may become a mother, then at  $x + 1$  become a mother again or not, independently of her previous experience. The accumulation of lifetime reproduction is the result of a random walk among these lives. Calculations of healthy life expectancy from prevalence data by the Sullivan method (Jagger et al. 2006) assume that an individual at age  $x$  might be a disabled nursing home resident, and at age  $x + 1$  a healthy marathon runner, and so on. The accumulation of healthy years is a random walk among the lives of individuals experiencing some specified levels of health and disability.

The common structure of all these calculations is that the only source of heterogeneity included as an  $i$ -state variable is age. The transitions between banker and factory worker, or between nursing home resident and marathon runner, seem strange because they call on our belief that age is *not* the only factor involved.

The resolution of the paradox would be, in each case, to create a multi-state model combining age and employment status, or age and health status, or age and parity as  $i$ -state variables. Such models can capture the dynamic dependence of the heterogeneity that leads to differences in rewards (see Rogers, Rogers, and Branch 1989 for a discussion of multistate models in health expectancy). When appropriate data are available, our analysis can be applied directly to them (e.g., Caswell 2014b for an age-parity model for fertility rewards).

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<sup>5</sup>We owe this delightful image to an anonymous reviewer.

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