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Reflexion

Two proofs of a recent formula by Griffith Feeney

Jutta Gampe

Anatoli Yashin

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Two proofs of a recent formula by Griffith Feeney

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Abstract

This reflexion provides two mathematical proofs for Equation (1) in Feeney (2006), published in this journal as 14-2.

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Comment from the Editor

In a paper on increments to life and mortality tempo that Feeney (2006) recently published in this journal, he gave the following decomposition of the difference between the expectations of life at birth for two cohorts,

$$e_0^c(t_2) - e_0^c(t_1) = -\int_0^\infty \lambda_c^{t_1,t_2}(x)\, d\ell_c(x,t_1)\,, \qquad (1)$$

but he did not give a mathematical proof. Since the correctness of this formula was contested during the reviewing process, this journal has decided to publish the following brief proofs given by two of our collaborators. The proofs are equivalent, but they look different and each may be useful for a different group of readers.

Proof by Jutta Gampe

The function $\lambda(x)$ described by Feeney, is given formally as

$$\lambda(x) = l_2^{-1}[l_1(x)] - x \tag{2}$$

when we drop some obvious subscripts and implicitly assume that everything is invertible etc. Equation (1) can be written either as

$$e_0^2 - e_0^1 = \int_0^\infty \{l_2(x) - l_1(x)\} dx$$

or as

$$= \int_0^1 \{l_2^{-1}(p) - l_1^{-1}(p)\} dp \ .$$

A change of variables $p \rightarrow l_1(x)$ leads to

$$\int_0^\infty \{l_2^{-1}[l_1(x)] - x\} f_1(x) dx = \int_0^\infty \lambda(x) f_1(x) dx \ ,$$

with $dl_1(x)/dx = -f_1(x)$. The latter integral equals

$$-\int_0^\infty \lambda(x) dl_1(x) \ .$$

Proof by Anatoli Yashin

The condition $l_2(x + \lambda(x)) = l_1(x)$ is equivalent to the condition that the random variables T_2 and $T_1 + \lambda(T_1)$ are identically distributed, hence $ET_2 = ET_1 + E\lambda(T_1)$, which is equation (1).

Reference

Feeney, Griffith (2006). Increments to Life and Mortality Tempo. *Demographic Research*, Volume 14, Article 2, online http://www.demographic-research.org/volumes/vol14/2/.