

Supplementary material

Smooth Constrained Mortality Forecasting

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Introduction

In the following document, in order to further assess the performance of the suggested *CP*-spline model, we perform an out-of-sample forecast exercise and test model stability by changing the time-window over which the model is estimated.

In Section A, we compare *CP*-splines with other five alternative forecasting methods. We perform this study on 8 populations. Effects of the change in time-windows (Sec. B) are presented for the suggested model and for a variant of the model proposed by Hyndman and Ullah (2007) which was identified as the next best performing model.

Finally, in Section C, we evaluate the impact and consequences of the choice of level of confidence on future rate-of-change over time. In other words, we check the effects of either reducing or increasing the 50% confidence interval we adopted for computing $\delta_L^{t_1}$ and $\delta_U^{t_1}$ in Section 3.2 of the paper.

A Comparison and out-of-sample forecast

In this section, we used mortality data for four countries (USA, Denmark, Japan and France) for both males and females. Data were taken from the Human Mortality Database (2019). We estimate six alternative and comparable forecasting methods:

- the proposed *CP*-splines: *CP*-S
- the smooth Lee-Carter variant by Delwarde et al. (2007): LCsmo
- the Lee-Carter variant by Lee and Miller (2001): LM
- the Lee-Carter variant by Booth et al. (2002): BMS
- the functional data approach by Hyndman and Ullah (2007): HU

- a robust version of the HU model: HUrob

We selected these approaches for two main reasons: they all model mortality in a single population by single year of age, and routines for estimating these models are freely available (e.g. Hyndman et al., 2019).

We first model all populations over the period 1960-2016 and we forecast up to 2050. The first three columns in Table A.1 and A.2 presents outcomes in terms of accuracy to model observed data. Goodness-of-fit is measured by Deviance (Dev, McCullagh and Nelder, 1989) and effective dimension (ED) gives a measures of the complexity of the model. The Bayesian Information Criterion (BIC) balances these two statistics and it allows us to evaluate precision in describing data. In all instances, *CP*-splines outperforms its competitors.

Based on previous estimations, we present life expectancy at birth (e_0) and average number of life years lost at birth (e_0^\dagger) in 2050. See associated columns in Table A.1 and A.2. For females, outcomes in e_0 are similar with exception of HU: this model seems more pessimistic for Denmark, Japan and France. Results for males life expectancy in 2050 are similar for all models. Lifespan disparity in 2050 is more model-dependent.

We assess accuracy and performance of the all six models by an out-of-sample forecast against the observed trends in life expectancy at birth (e_0), average number of life years lost at birth (e_0^\dagger) and log-mortality over all ages and years (η). Specifically, we fit all models from 1960 to 2006, and we forecast mortality 10 years ahead (2007-2016), comparing these forecast values to those observed over that decade.

We measure the accuracy of the models by computing mean absolute error (MAE), root mean square error (RMSE) and mean error (ME). Whereas ME describes the forecast bias due to the model, the other measures assess the performance of the model on the same units as the measured variable (Chatfield, 2000).

Right panels in Table A.1 and A.2 present the outcomes of this test. Given 4 populations, 2 sexes, 3 accuracy measures and 3 demographic indicators, we compare six alternative approaches over 72 values: the proposed *CP*-spline model outperforms its competitors 39 times (54%), followed by HU and HUrob with 16 and 8 times, respectively.

To give a graphical overview of these results, Figure A.1 shows three selected ages over time for US males mortality data from the presented out-of-sample forecast. This plot gives an immediate picture of the goodness-of-fit and forecasting reliability for both *CP*-spline and Hyndman-Ullah approach. To ensure a fair comparison, a residual bootstrap approach is applied to acquire predictive intervals on both models. In particular, the Hyndman-Ullah merely extrapolates a linear pattern which is already inaccurate to describe the mortality patterns. On the contrary, the proposed *CP*-spline approach is able to fully capture observed mortality developments, including erratic trends in the

latest years.

Finally, an out-of-sample forecast exercise as presented in this section could also be performed to set the confidence interval used to compute $\delta_L^{t_1}$ and $\delta_U^{t_1}$ (cf. Section C).

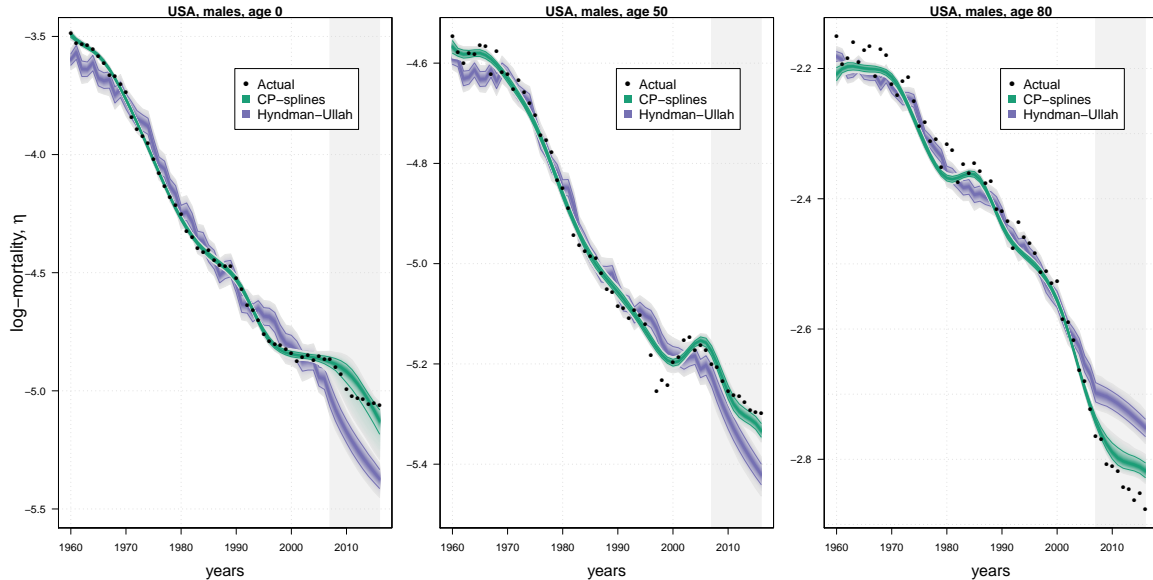


Figure A.1: Illustration of the outcomes from the out-of-sample forecast exercise. Empirical, model and forecast mortality along with their bootstrapped distributions for selected ages (0, 50 and 80) over years. The *CP*-spline approach is compared with a variant of the Hyndman-Ullah model. USA males, ages 0-105. Fitting period 1960-2006, forecast up to 2016 and compared to observed values in 2007-2016.

B Changing the time-window

To test the stability of the *CP*-splines and of the second classified model by Hyndman and Ullah (2007), we apply both of them to different periods of time. We select progressively smaller data periods, going from t_0 to 2016. We start with a value of t_0 equal to 1935 and end with t_0 equal to 1985. The models applied to different time-windows are used to produce forecasts of life expectancy at birth and e_0^\dagger in the year 2050, for both USA males and Danish females. Results are plotted in Figure B.1.

Whereas life expectancy at birth for Danish females is barely affected by the changes in the modelled periods when forecast is obtained by *CP*-splines, outcomes from Hyndman-Ullah model are very erratic and time-window dependent. The observed intervals of e_0 in 2050 obtained by changing the modelled periods are $[86.33, 87.26]$ for the *CP*-splines and $[84.57, 87.71]$ years for the Hyndman-Ullah model. For US males, the values of life expectancy at birth in 2050 are sensitive to the time-windows used in modelling the data, for both models, although outcomes for the Hyndman-Ullah model is more sensible to inclusion of years during World War II. The difference between the largest

and smallest value of e_0 estimated by different time-windows is 2.5 (2.8) years for the CP -spline (Hyndman-Ullah) model.

Moreover, for USA males, the Hyndman-Ullah tends to regularly provide higher values than those obtained by the CP -spline model when earlier years are included. This feature is likely due to the ability of the proposed method to capture recent mortality stagnation among USA males, as portrayed in the middle panel of Figure A.1 and Figure 9 in the paper. On the contrary, although Hyndman-Ullah model generalizes a Lee-Carter model using six principal components to describe mortality development, it is still too rigid to incorporate further fluctuations in its forecasts.

Concerning lifespan variability, the proposed model gives a rather constant value of e_0^\dagger in 2050, whatever the modelled period and for both datasets. Conversely, the values of e_0^\dagger in 2050 obtained by the Hyndman-Ullah model are highly sensitive to the choice of the time-window selected to model the data, with peaks and troughs over t_0 .

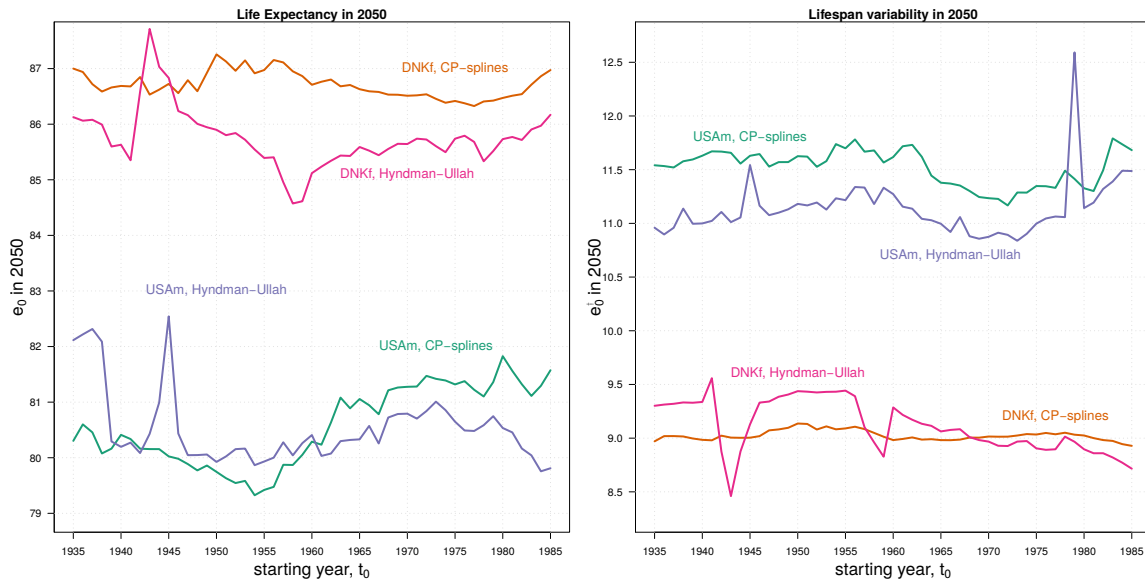


Figure B.1: Illustration of the outcomes from different time-windows for modelling the data. Life expectancy at birth (left panel) and a measure of lifespan variability (e_0^\dagger , right panel) in 2050. Modelled period t_0 to 2016. Year t_0 given on horizontal axis. The proposed CP -spline approach is compared with a variant of the Hyndman-Ullah model. USA males and Denmark females, ages 0-105.

C Confidence level in rate-of-change over time

In Section 3.2 of the paper we suggest using a 50% confidence interval of the smooth rate-of-change over time to compute $\delta_L^{t_1}$ and $\delta_U^{t_1}$. In other words, we constrain future mortality at each age to lie within the interquartile range of the observed mortality improvement. In this section we assess the consequences of this choice, showing that larger percentages

may lead to unreasonable outcomes, especially when large fluctuations are observed in the past. We already pointed out in the paper that extremely small values for computing $\delta_L^{t_1}$ and $\delta_U^{t_1}$ leads to an excessively rigid model, similar to a Lee-Carter model. Moreover we perform an out-of-sample forecast exercise to assess the robustness of our choice as well as to provide a solid base for selecting eventual population-specific constraints.

Figure C.1 presents the results for the two illustrative datasets when different confidence intervals are used to compute $\delta_L^{t_1}$ and $\delta_U^{t_1}$. In the top panels, we show both life expectancy and a measure of lifespan variability (e_0^\dagger). The time trend for a specific age (35) is portrayed in the bottom panels for Danish females (left) and USA males (right). We select this age since it displays relatively large fluctuations for USA males and therefore differences in changing confidence levels are more visible.

It is clear that for Danish females, selecting different percentages only slightly changes future trends in summary measures and log-mortality. On the other hand, future mortality development for USA males depends upon the choice of confidence interval for computing $\delta_L^{t_1}$ and $\delta_U^{t_1}$. Whereas values below 50% produce similar outcomes in terms of e_0 and e_0^\dagger , allowing a high degree of flexibility in future mortality rate-of-change leads to unreasonably low mortality rates (see purple and green lines in the bottom-right panel), and consequently high future life expectancy and an odd pattern of lifespan variability over a short-term forecasting horizon. This is a consequence of the highly erratic pattern of US mortality in the recent decades. Taking age 35 as an example, allowing future rate-of-change to lie between the 95% confidence intervals of past rate-of-change means that the rapid mortality improvements experienced by US males in the late 1990s would be conceivable for future years. However this fast improvement at age 35 was due mainly to the introduction of antiretroviral therapies to treat HIV-AIDS. Meanwhile, by choosing 50%, we also consider as implausible for the future a mortality increase such as that experienced by US males at age 35 in the 80s due to the HIV-AIDS epidemic.

Table C.1 presents an additional out-of-sample forecast exercise. Following the same design presented in Section A, we compute the root mean square error (RMSE) for CP -splines with different percentages for computing $\delta_L^{t_1}$ and $\delta_U^{t_1}$ as well as for Hyndman-Ullah and its robust version approaches. These two models were identified as the next best performing models in the previous out-of-sample forecast exercise. We can read outcomes in Table C.1 from two different perspectives. Firstly, CP -splines outperform alternative methods in most of the cases: 18 out of 24 measures if we fix the percentage equal to 50% and 22 out of 24 measures if we allow to modify the percentage. Secondly, this exercise shows that different percentages for computing $\delta_L^{t_1}$ and $\delta_U^{t_1}$ could be selected for different populations based on measurable evidences. For instance it would be reasonably better to allow Danish females future age-specific rate-of-change to lay within the 95% confidence interval of the smooth observed time-trends. However, by looking at the RMSE for the

log-mortality over age and time (η), differences with the proposed 50% are not large, as already suggested from the outcomes in Figure C.1. Obviously by changing target measure (e_0 , e_0^\dagger and η) and/or time-window for the out-of-sample forecast, one could select different percentage and obtain distinct forecast values. The magnitude of these differences (e.g. the importance of the selected percentage) could be eventually assessed by an out-of-sample forecast exercise as presented here.

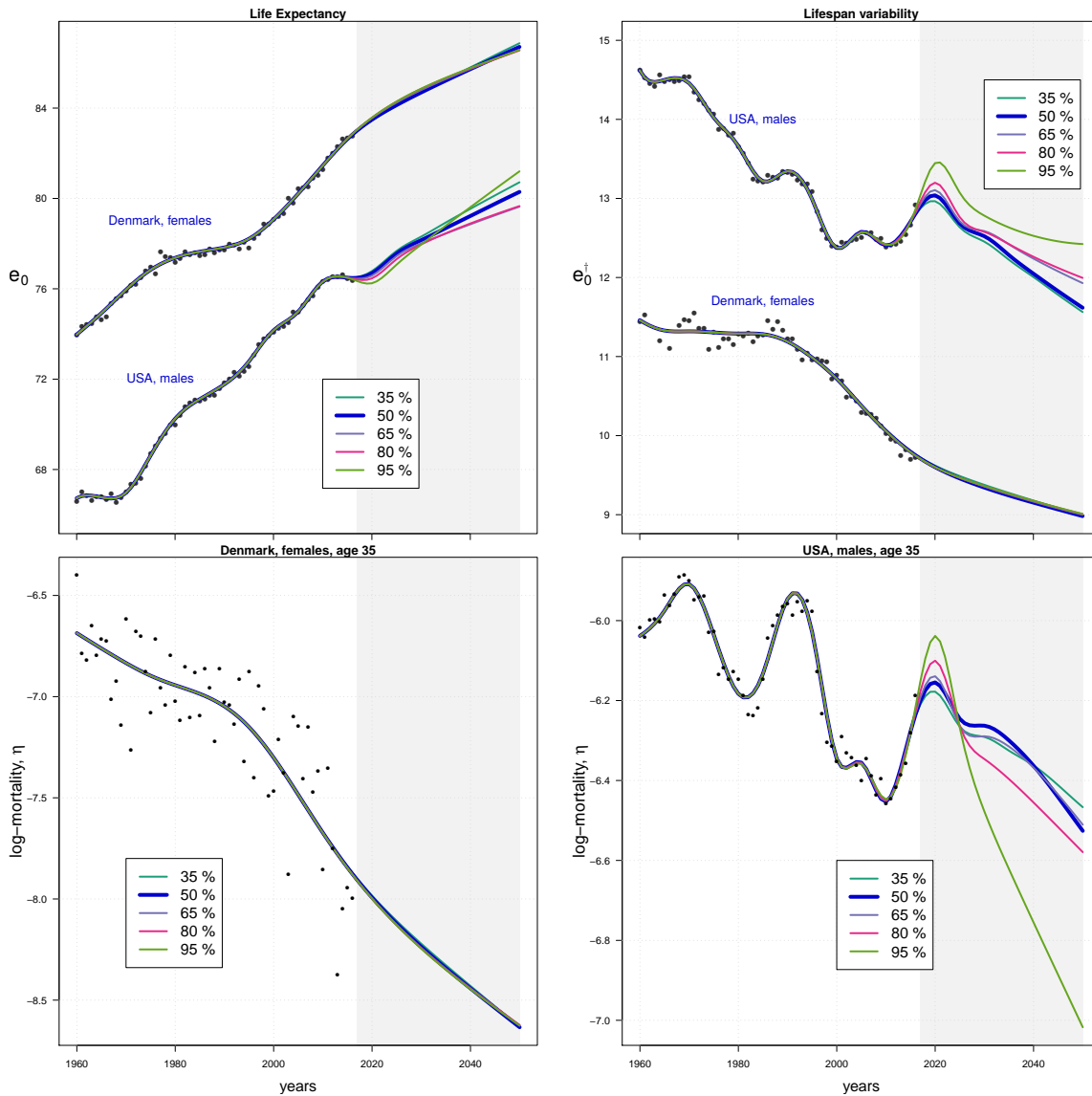


Figure C.1: Illustration of the outcomes of different levels of confidence for computing $\delta_L^{t_1}$ and $\delta_U^{t_1}$. We used 35, 50, 65, 80 and 95% confidence intervals from the smooth observed rate-of-change over time. Top panels: Life expectancy at birth (left) and a measure of lifespan variability (e_0^\dagger , right). Bottom panels: death rates for age 35 over years. CP -spline model on Danish females and US males, ages 0-105, years 1960-2016, forecast up to 2050.

References

- Booth, H., J. Maindonald, and L. Smith (2002). Applying Lee-Carter under conditions of variable mortality decline. *Population Studies* 56, 325–336.
- Chatfield, C. (2000). *Time-series forecasting*. CRC Press.
- Delwarde, A., M. Denuit, and P. H. C. Eilers (2007). Smoothing the Lee-Carter and Poisson log-bilinear models for mortality forecasting: A penalized log-likelihood approach. *Statistical Modelling* 7, 29–48.
- Human Mortality Database (2019). *University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany)*. Available at www.mortality.org. Data downloaded on July 2019.
- Hyndman, R. J., H. Booth, L. Tickle, and J. Maindonald (2019). *demography: Forecasting Mortality, Fertility, Migration and Population Data*. . R package version 1.22.
- Hyndman, R. J. and M. S. Ullah (2007). Robust forecasting of mortality and fertility rates: A functional data approach. *Computational Statistics & Data Analysis* 51, 4942–4956.
- Lee, R. D. and T. Miller (2001). Evaluating the Performance of the Lee-Carter Method for Forecasting Mortality. *Demography* 38, 537–549.
- McCullagh, P. and J. A. Nelder (1989). *Generalized Linear Models* (2nd ed.). Monographs on Statistics Applied Probability. London: Chapman & Hall.

Table A.1: Numerical outcomes from the estimated and forecast models as well as for the out-of-sample forecast exercise. Females. Models estimated over years 1960-2016, ages 0-105 and forecast up to 2050. Deviance (Dev), Effective Dimension (ED) and Bayesian Information Criterion (BIC) are presented to compare performances of the models in the years 1960-2016. Lower values of BIC (in bold) indicates a better fit. Values for e_0 , e_0^f in 2050 are also provided. Out-of-sample forecast is carried out with fitting period 1960-2006, forecast up to 2016 and compared to observed values in 2007-2016. Models are compared using Mean absolute error (MAE), root mean square error (RMSE) and mean error (ME). Accuracy measures are computed on e_0 , e_0^f and η . Lower values of the MAE and the RMSE, and ME closer to zero (in bold) correspond to greater accuracy. Populations: USA, Denmark, Japan and France, females. Models: *CP-S*, smooth Lee-Carter (LCsmo), Lee-Miller (LM), Booth-Maindonald-Smith (BMS), Hyndman-Ullah (HU) and a robust version of Hyndman-Ullah (HUrob).

	Observed years + Forecast				Out-of-sample: 1960-2006 → 2007-2016										
	Dev	ED	BIC	e_0	e_0^f	MAE	RMSE	ME	MAE	RMSE	ME	MAE	RMSE	ME	
USAf	<i>CP-S</i>	27988	259	30242	84.53	10.40	0.116	0.154	-0.054	0.148	0.224	0.131	0.051	0.079	0.004
	LCsmo	93283	251	95466	85.37	9.71	0.188	0.203	0.117	0.158	0.229	0.141	0.097	0.128	-0.011
	LM	97147	267	99471	85.07	9.82	0.164	0.184	0.080	0.140	0.204	0.108	0.092	0.123	0.002
	BMS	94125	267	96450	85.38	9.71	0.184	0.200	0.110	0.139	0.200	0.099	0.092	0.123	-0.001
DNKf	HU	25559	1072	34893	83.80	9.99	0.347	0.369	0.347	0.242	0.313	0.242	0.082	0.103	-0.020
	HUrob	24860	1072	34193	84.03	10.26	0.274	0.302	0.274	0.207	0.276	0.205	0.065	0.086	-0.022
	<i>CP-S</i>	6854	100	7728	86.97	9.07	0.597	0.690	0.597	0.212	0.235	-0.212	0.201	0.324	-0.106
	LCsmo	11771	122	12831	86.71	9.10	1.343	1.420	1.343	0.832	0.850	-0.832	0.293	0.420	-0.245
JPNf	LM	12623	267	14947	86.83	9.20	0.673	0.793	0.652	0.636	0.658	-0.636	0.261	0.367	-0.065
	BMS	11869	267	14193	86.75	9.21	1.213	1.299	1.213	0.688	0.708	-0.688	0.264	0.373	-0.142
	HU	6999	1072	16331	85.12	9.29	1.086	1.237	1.086	0.482	0.516	-0.482	0.236	0.344	-0.145
	HUrob	6931	1072	16262	85.46	9.09	0.892	1.033	0.887	0.213	0.248	-0.213	0.213	0.328	-0.124
FRAf	<i>CP-S</i>	15056	229	17050	93.52	8.32	0.339	0.404	-0.314	0.089	0.105	-0.053	0.079	0.121	0.054
	LCsmo	69939	241	72040	93.77	7.21	0.979	1.049	-0.979	0.306	0.317	0.306	0.226	0.287	0.222
	LM	84235	267	86560	93.81	7.28	0.681	0.777	-0.681	0.398	0.405	0.398	0.188	0.241	0.174
	BMS	74952	267	77276	93.76	7.30	0.918	0.986	-0.918	0.448	0.455	0.448	0.213	0.265	0.204
FRAf	HU	11376	1072	20709	90.70	7.93	0.263	0.319	-0.230	0.266	0.298	-0.266	0.082	0.123	-0.004
	HUrob	11127	1072	20460	90.82	7.89	0.527	0.601	-0.527	0.110	0.126	-0.084	0.086	0.122	0.024
	<i>CP-S</i>	13803	217	15692	90.01	8.63	0.324	0.368	-0.314	0.034	0.047	-0.019	0.078	0.107	0.020
	LCsmo	22577	172	24072	90.28	8.02	0.384	0.431	-0.384	0.258	0.263	0.258	0.131	0.180	-0.023
HUrob	LM	25426	267	27750	90.27	8.04	0.302	0.358	-0.298	0.238	0.244	0.238	0.115	0.157	-0.016
	BMS	23626	267	25951	90.31	8.03	0.392	0.438	-0.392	0.262	0.267	0.262	0.116	0.157	-0.006
	HU	9113	1072	18446	89.71	8.30	0.134	0.172	0.081	0.094	0.101	0.091	0.085	0.120	-0.019
	HUrob	8807	1072	18140	89.66	8.35	0.208	0.258	-0.173	0.039	0.048	0.017	0.077	0.109	-0.002

Table A.2: Numerical outcomes from the estimated and forecast models as well as for the out-of-sample forecast exercise. Males. Models estimated over years 1960-2016, ages 0-105 and forecast up to 2050. Deviance (Dev), Effective Dimension (ED) and Bayesian Information Criterion (BIC) are presented to compare performances of the models in the years 1960-2016. Lower values of BIC (in bold) indicates a better fit. Values for e_0 , e_0^\dagger in 2050 are also provided. Out-of-sample forecast is carried out with fitting period 1960-2006, forecast up to 2016 and compared to observed values in 2007-2016. Models are compared using Mean absolute error (MAE), root mean square error (RMSE) and mean error (ME). Accuracy measures are computed on e_0 , e_0^\dagger and η . Lower values of the MAE and the RMSE, and ME closer to zero (in bold) correspond to greater accuracy. Populations: USA, Denmark, Japan and France, males. Models: *CP-S*, smooth Lee-Carter (LCsmo), Lee-Miller (LM), Booth-Maindonald-Smith (BMS), Hyndman-Ullah (HU) and a robust version of Hyndman-Ullah (HUrob).

	Observed years + Forecast						Out-of-sample: 1960-2006 \rightarrow 2007-2016								
	1960-2016			2050			e_0			e_0^\dagger			η		
	Dev	ED	BIC	e_0	e_0^\dagger		MAE	RMSE	ME	MAE	RMSE	ME	MAE	RMSE	ME
USAm	<i>CP-S</i>	32906	306	35573	80.36	11.69	0.131	0.192	-0.034	0.238	0.349	0.232	0.064	0.092	-0.005
	LCsmo	175726	222	177663	81.28	10.81	0.231	0.269	0.082	0.558	0.614	0.558	0.105	0.132	-0.025
	LM	186854	267	189179	80.94	10.83	0.271	0.297	0.152	0.512	0.570	0.512	0.101	0.126	-0.025
	BMS	181019	267	183344	81.26	10.72	0.213	0.264	0.048	0.544	0.600	0.544	0.102	0.128	-0.017
	HU	30531	1072	39865	80.41	11.28	0.277	0.306	0.205	0.350	0.469	0.35	0.084	0.110	-0.030
	HUrob	29179	1072	38513	80.40	11.06	0.273	0.304	0.193	0.296	0.399	0.296	0.072	0.099	-0.019
DNKIm	<i>CP-S</i>	7178	111	8142	83.25	9.61	0.940	1.069	0.940	0.273	0.308	-0.254	0.221	0.358	-0.158
	LCsmo	11461	126	12557	82.70	9.26	1.270	1.440	1.270	0.222	0.247	-0.164	0.260	0.372	-0.149
	LM	11648	267	13972	82.78	9.90	1.286	1.457	1.286	0.364	0.409	-0.354	0.294	0.427	-0.075
	BMS	11589	267	13912	82.82	9.90	3.722	3.804	3.722	0.834	0.861	-0.834	0.515	0.809	-0.495
	HU	7637	1072	16965	83.41	8.38	0.942	1.047	0.943	0.076	0.097	0.039	0.229	0.364	-0.164
	HUrob	7664	1072	16993	84.97	8.50	1.170	1.345	1.170	0.225	0.250	-0.184	0.238	0.373	-0.181
JPNm	<i>CP-S</i>	17186	234	19224	87.05	9.28	0.107	0.152	-0.042	0.082	0.095	-0.060	0.070	0.104	0.033
	LCsmo	49355	244	51479	87.34	8.78	0.357	0.391	-0.357	0.081	0.088	0.001	0.11	0.154	0.064
	LM	52449	267	54774	87.59	9.42	0.266	0.311	-0.266	0.083	0.092	0.023	0.106	0.163	0.064
	BMS	51444	267	53769	87.50	9.42	0.355	0.389	-0.355	0.084	0.098	0.037	0.11	0.168	0.074
	HU	18869	1072	28201	87.07	9.07	0.176	0.226	-0.156	0.242	0.26	-0.242	0.072	0.101	0.023
	HUrob	19018	1072	28350	87.46	8.97	0.250	0.300	-0.250	0.168	0.196	-0.168	0.068	0.097	0.030
FRAm	<i>CP-S</i>	13018	223	14963	83.85	9.93	0.224	0.297	0.217	0.067	0.081	-0.067	0.073	0.101	-0.010
	LCsmo	42241	184	43841	84.66	9.72	0.421	0.463	0.421	0.121	0.130	-0.121	0.152	0.209	-0.081
	LM	46071	267	48396	84.81	9.67	0.243	0.305	0.243	0.077	0.089	-0.077	0.174	0.455	-0.101
	BMS	45201	267	47525	84.77	9.68	0.587	0.617	0.587	0.153	0.160	-0.153	0.177	0.433	-0.129
	HU	11439	1072	20772	84.15	9.67	0.132	0.166	0.009	0.192	0.205	0.192	0.094	0.134	0.001
	HUrob	10932	1072	20265	84.29	9.72	0.469	0.540	-0.469	0.045	0.053	0.031	0.094	0.138	0.049

Table C.1: Numerical outcomes from an out-of-sample forecast exercise aiming to check robustness with respect to the choice in the confidence interval for the time-trend constraints.

Models estimated with fitting period 1960-2006, forecast up to 2016 and compared to observed values in 2007-2016. Models are compared using root mean square error (RMSE). Accuracy measures are computed on e_0 , e_0^\dagger and $\boldsymbol{\eta}$. Lower values of the RMSE (in bold) correspond to greater accuracy. Populations: USA, Denmark, Japan and France, females and males. Models: CP -splines with different percentage in computing $\delta_L^{t_1}$ and $\delta_U^{t_1}$, Hyndman-Ullah (HU) and its robust version (HUrob).

		CP -splines					HU	HUrob
		35%	50%	65%	80%	95%		
USAf	e_0	0.128	0.154	0.234	0.279	0.313	0.369	0.302
	e_0^\dagger	0.214	0.224	0.247	0.268	0.263	0.313	0.276
	$\boldsymbol{\eta}$	0.076	0.079	0.083	0.089	0.093	0.103	0.086
DNKf	e_0	0.810	0.690	0.555	0.432	0.375	1.237	1.033
	e_0^\dagger	0.299	0.235	0.159	0.102	0.080	0.516	0.248
	$\boldsymbol{\eta}$	0.332	0.324	0.319	0.317	0.316	0.344	0.328
JPNf	e_0	0.480	0.404	0.318	0.244	0.179	0.319	0.601
	e_0^\dagger	0.086	0.105	0.142	0.178	0.158	0.298	0.126
	$\boldsymbol{\eta}$	0.129	0.121	0.113	0.111	0.110	0.123	0.122
FRAf	e_0	0.310	0.368	0.436	0.493	0.751	0.172	0.258
	e_0^\dagger	0.046	0.047	0.049	0.052	0.150	0.101	0.048
	$\boldsymbol{\eta}$	0.108	0.107	0.108	0.111	0.126	0.120	0.109
USAm	e_0	0.216	0.192	0.146	0.165	0.250	0.306	0.304
	e_0^\dagger	0.348	0.349	0.333	0.349	0.371	0.469	0.399
	$\boldsymbol{\eta}$	0.092	0.092	0.091	0.101	0.117	0.110	0.099
DNKm	e_0	1.336	1.069	0.824	0.748	0.714	1.047	1.345
	e_0^\dagger	0.354	0.308	0.294	0.295	0.298	0.097	0.250
	$\boldsymbol{\eta}$	0.368	0.358	0.352	0.350	0.348	0.364	0.373
JPNm	e_0	0.167	0.152	0.153	0.220	0.326	0.226	0.300
	e_0^\dagger	0.088	0.095	0.112	0.126	0.105	0.260	0.196
	$\boldsymbol{\eta}$	0.104	0.104	0.101	0.095	0.093	0.101	0.097
FRAm	e_0	0.391	0.297	0.159	0.161	0.334	0.166	0.540
	e_0^\dagger	0.106	0.081	0.049	0.043	0.052	0.205	0.053
	$\boldsymbol{\eta}$	0.108	0.101	0.098	0.103	0.112	0.134	0.138