Multistate life expectancy estimates using racial transition probabilities for 5-year intervals (instead of 9-months)

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1 Background

Muniz et al. [2024] used transition probabilities between the white, brown and black racial categories (*branco/a, pardo/a, preto/a* in Portuguese) based on longitudinal data from PNAD-C conducted by the Brazilian Institute of Geography and Statistics (IBGE) between 2017 and 2019. The same individuals self-reported their race at two points in time, nine months apart. By cross-tabulating these data by age, we obtained the proportions of people who reclassified their race over a nine-month period. The calculations in our original article assumed that the racial reclassification proportions would remain constant over five years. Subsequent reviewer feedback on a related project highlighted the implications of this simplifying assumption, particularly its effects on life expectancies by race and for the overall population.

In response, we present alternative estimates that allow the nine-month probabilities to compound across the five-year period. This approach is intended to prompt researchers to consider how temporal stability—or instability—in reclassification patterns may influence results in future applications of this method. Importantly, both estimation strategies have distinct strengths and limitations[1], and we interpret them as providing plausible lower- and upper-end estimates relative to a scenario in which repeated racial classification data are available throughout the full five-year period.

To extend our original reclassification probabilities from nine months to five-years, we apply the Matrix Diagonalization Method, a Markovian mathematical technique that enables the conversion of transition probabilities across different time intervals [Kitsul and Philipov, 1982, Rees, 1977]. For example, in the first age group (starting at age 0 and before combining them with race-specific probabilities of dying) the nine-month (0.75-year) transition matrix of racial reclassification probabilities is:

[1] On one hand, extrapolating nine-month probabilities to a five-year period may introduce biases, potentially overestimating transitions, as transition rates typically decline over longer durations and the method does not explicitly account for multiple transitions within the period. On the other hand, if the observed nine-month probabilities already reflect a stable pattern, this approach may provide a more reasonable adjustment than assuming constant transition probabilities over five years.

$$\mathbf{\Pi}(0,.75) = \begin{pmatrix} .8647 & .1308 & .0045 \\ .0903 & .8828 & .0269 \\ .0342 & .2663 & .6994 \end{pmatrix} \approx \mathbf{W}\mathbf{\Lambda}\mathbf{W}^{-1} \tag{1}$$

where $\mathbf{W}$ is the matrix whose columns are the eigenvectors of $\mathbf{\Pi}(0,.75)$, $\mathbf{\Lambda}$ is a diagonal matrix containing the eigenvalues of $\mathbf{\Pi}(0,.75)$, and $\mathbf{W}^{-1}$ is the inverse of $\mathbf{W}$. The first row of the transition matrix in (1) indicates that over a nine-month period, 86.47% of individuals initially classified as White remain White, 13.08% transition to Brown, and 0.45% transition to Black. The second and third rows represent the transition probabilities for individuals initially classified as Brown and Black, respectively. For instance, 3.42% of individuals initially classified as Black transition to White, 26.63% to Brown, and 69.94% remain Black over the nine-month period. By diagonalizing (1), we obtain the five-year transition matrix using

$$\mathbf{\Pi}(0,5) \approx \mathbf{W}\mathbf{\Lambda}^{5/.75}\mathbf{W}^{-1} = \begin{pmatrix} .5051 & .4585 & .0363 \\ .3174 & .6217 & .0609 \\ .2557 & .6075 & .1367 \end{pmatrix} \tag{2}$$

This conversion method is systematically applied across all age groups, ensuring a consistent estimation of racial reclassification dynamics over a five-year period. To estimate the final probabilities of transitioning between racial states or to death, we integrate race-specific mortality probabilities from Table 1 [Muniz et al., 2024, 462] with survivorship proportions derived from (2), adjusting them to sum to 1. This process leads to the following revision of our original Table 2:

Table 2A: Probabilities of surviving ($\tilde{\pi}_x^{i,i}$), reclassifying to another racial category ($\tilde{\pi}_x^{i,j}$), or dying ($\tilde{q}_x^i$) by age group and racial category adjusted to sum to 1 for a 5-year period

	1. White				**2. Brown**				**3. Black**			
Age group	$\tilde{\pi}^{1,W}$	$\tilde{\pi}^{1,Br}$	$\tilde{\pi}^{1,B}$	$\tilde{q}^1$	$\tilde{\pi}^{2,W}$	$\tilde{\pi}^{2,Br}$	$\tilde{\pi}^{2,B}$	$\tilde{q}^2$	$\tilde{\pi}^{3,W}$	$\tilde{\pi}^{3,Br}$	$\tilde{\pi}^{3,B}$	$\tilde{q}^3$
0-4	0.4996	0.4535	0.0359	0.0110	0.3119	0.6109	0.0599	0.0174	0.2537	0.6027	0.1356	0.0080
5-9	0.4693	0.4880	0.0418	0.0009	0.2801	0.6506	0.0678	0.0015	0.2256	0.6407	0.1325	0.0012
10-14	0.4558	0.4968	0.0462	0.0011	0.2665	0.6564	0.0748	0.0023	0.2156	0.6446	0.1375	0.0023
15-19	0.4580	0.4873	0.0510	0.0037	0.2663	0.6397	0.0835	0.0105	0.2108	0.6297	0.1485	0.0110
20-24	0.4891	0.4527	0.0527	0.0055	0.2746	0.6213	0.0908	0.0133	0.2120	0.6021	0.1714	0.0145
25-29	0.5233	0.4205	0.0505	0.0057	0.2823	0.6126	0.0929	0.0121	0.2158	0.5844	0.1864	0.0135
30-34	0.5243	0.4163	0.0529	0.0066	0.2758	0.6140	0.0971	0.0131	0.2110	0.5799	0.1943	0.0148
35-39	0.5346	0.4050	0.0521	0.0083	0.2847	0.6018	0.0973	0.0162	0.2165	0.5750	0.1900	0.0186
40-44	0.5329	0.4018	0.0535	0.0118	0.2818	0.5961	0.1006	0.0215	0.2108	0.5649	0.1989	0.0253
45-49	0.5420	0.3875	0.0530	0.0175	0.2903	0.5789	0.1011	0.0296	0.2171	0.5548	0.1921	0.0360
50-54	0.5644	0.3594	0.0496	0.0265	0.2952	0.5628	0.1007	0.0413	0.2211	0.5367	0.1901	0.0521
55-59	0.5681	0.3448	0.0466	0.0406	0.2969	0.5482	0.0964	0.0585	0.2194	0.5261	0.1808	0.0737
60-64	0.5665	0.3269	0.0442	0.0624	0.2979	0.5237	0.0942	0.0842	0.2147	0.4974	0.1841	0.1038
65-69	0.5434	0.3178	0.0437	0.0951	0.2912	0.4951	0.0918	0.1219	0.2065	0.4674	0.1822	0.1438
70-74	0.5323	0.2809	0.0394	0.1474	0.2882	0.4504	0.0842	0.1772	0.2103	0.4339	0.1542	0.2016
75-79	0.4643	0.2654	0.0374	0.2329	0.2690	0.3924	0.0756	0.2630	0.1993	0.3828	0.1370	0.2809
80+	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000

Notes: W = White; Br = Brown; B = Black. Subscripts n and x are omitted from $\tilde{\pi}^{i,j}$ and $\tilde{q}^i$ to avoid cluttering. Mortality estimates at young ages may be affected by differential underregistration of infant deaths by race/color, which is likely higher among Black and Brown populations (Caldas et al. [2017], Cardoso et al. [2005]).

Source: Race-specific probabilities of dying are based on population data from the 2010 Brazilian census, from the third quarter of the 2020 PNAD-C microdata, and the average annual number of deaths between 2010 and 2019. Enumerated populations were corrected for under coverage, baseline racial reclassification, and international migration. Mortality estimates were produced using **ilt**, a Stata program developed by Muniz [2023].

There are multiple ways to generate five-year transition matrices from short-interval observations. In the original article, we adopted the most direct extrapolation—treating nine-month transition probabilities as constant over five years—which leads to lower total life expectancy. The diagonalization method presented here represents a distinct alternative, in which five-year transitions result from repeated compounding of short-interval mobility. Each method therefore imposes strong but different assumptions: one freezes short-interval mobility; the other assumes Markovian homogeneity and allows repeated transitions within the interval.

2 Diagonalization results

Table 2A produces a set of life expectancy estimates and summary measures, which are presented below in analogous format to the original article.

Figure 1A: Population-based (unconditional) life expectancy by age and racial classification

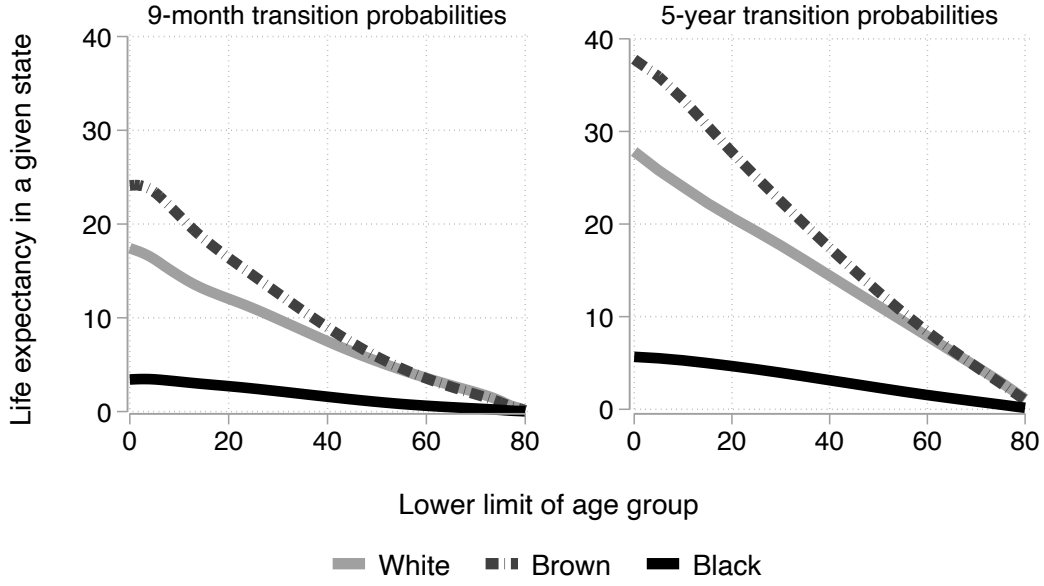

Figure 1A shows that, under the alternate 5-year transition framework, life expectancy in each racial state rises modestly relative to the constant nine-month assumption, but racial trajectories remain substantively similar. The diagonalization approach permits multiple transitions within each five-year window and therefore removes the implicit frequency constraint embedded in the original scaling assumption. This alternate multistep structure is closer to demographic practice for interval conversion, while still not representing the full reclassification complexity that future multi-wave racial measurement would allow.

Table 3A: Summary multistate measures

	State i			
	White	Brown	Black	Death
Proportion of life spent in state i	0.3433	0.4661	0.0699	0.1207
Probability of dying in state i	0.3180	0.4192	0.0833	-
Average duration of state i	11.4581	12.8695	6.0578	-
Mean age of persons in state i	38.7377	36.5258	41.0071	61.1277

Table 3A demonstrates that the proportion of life spent as Brown (0.466) continues to exceed that spent as White (0.343) or Black (0.070), a central empirical pattern that persists across both interval-construction approaches. Even though the revised 5-year probabilities increase overall survival time and reclassification opportunity, the relative ordering of the three racial states is unchanged, showing that the central structural conclusion does not depend on how five-year transitions are produced. This stability is crucial: while the diagonalized model produces higher cumulative time alive and therefore slightly higher time spent in each racial category, it does not overwrite the core narrative of asymmetric racial fluidity, only alters its magnitude. Thus, Table 3A provides the clearest evidence that the multistate racial classification process is robust to alternative scaling assumptions, even when absolute estimates shift.

Table 4A: Status-based (conditional) life expectancies at birth by origin state

	Expected number of years to be lived in each state		
State of origin (at age 0)	White	Brown	Black
White	29.597	25.76	25.603
Brown	36.405	39.634	37.485
Black	5.4872	5.6103	8.5796
Total	71.489	71.004	71.668

Table 4A further confirms that interval conversion affects levels rather than ordering. Life expectancy conditional on initial classification as Brown remains slightly above that of Whites and Blacks, while the alternative 5-year model yields the highest total longevity for individuals initially identifying as Black (71.7) because expanded transition space defers absorption into death and permits longer chains of cross-state mobility. As noted above, the original nine-month extrapolation produces a conservative lower estimate, whereas the diagonalized specification allows multi-step racial shifts and produces a higher estimate. Together, they bracket the true cumulative process and clarify that the

difference is assumption-driven rather than theory-driven. Future longitudinal designs with three or more race observations will allow relaxing both bounding assumptions entirely, directly addressing the question of which dynamic pattern is empirically most realistic.

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