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*Research Article*

### **Standardized mean age at death (MAD<sup>std</sup>): Exploring its potential as a measure of human longevity**

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## **Standardized mean age at death (MAD<sup>std</sup>): Exploring its potential as a measure of human longevity**

**Markus Sauerberg<sup>1</sup>**

**Marc Luy<sup>2</sup>**

### **Abstract**

#### **BACKGROUND**

Period mean age at death (MAD) is affected by a population's age structure, and therefore by its mortality, fertility, and migration history. Period life expectancy ( $e_0$ ) is also a mean age at death, for a standardized population with a stationary age structure. It depends only on current mortality rates. Here, we explore a middle ground: an age-standardized measure of period age at death, called MAD<sup>std</sup>, that includes both past and present mortality influences, while omitting the effects of past fertility and migration.

#### **OBJECTIVE**

We want to highlight the common structure of the three measures by expressing them as weighted averages with different weighting functions. This allows us to examine them from the perspective of compositional change; i.e., how changes in the underlying age structure affect MAD, MAD<sup>std</sup>, and  $e_0$ .

#### **METHODS**

We compare MAD<sup>std</sup> with  $e_0$  and MAD formally and empirically, using data on six countries from 1990 to 2020. A particular focus is given to the effect of the increased mortality in 2020 on the three longevity measures.

#### **RESULTS**

The  $e_0$  indicator gives a higher average age at death than MAD and MAD<sup>std</sup> because the relative number of older individuals is comparatively high in the hypothetical period life table population. While  $e_0$  declines between 2019 and 2020, both MAD and MAD<sup>std</sup> show increases in 2020. This can be attributed to differences in the dynamics of the age structures underlying the three indicators. Only the life table population shifts to younger ages, whereas for the observed population and standardized population in 2020 the relative numbers of older individuals increased.

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## CONCLUSION

Trends in MAD and MAD<sup>std</sup> are less sensitive to recent developments in mortality, making  $e_0$  the most valuable for examining changes in period mortality rates over time. Considering the interaction between changes in age-specific mortality rates and changes in the underlying age structure deepens the understanding of diverging time trends in MAD, MAD<sup>std</sup>, and  $e_0$ .

## CONTRIBUTION

We use the formulas developed by Vaupel and Canudas-Romo (2002) to study the change in all three measures over time. Formulas provided by Vaupel and Zhang (2012) are used to study cross-sectional differences in MAD, MAD<sup>std</sup>, and  $e_0$ . These help us to better understand the differences between the longevity measures and their most appropriate applications.

## 1. Introduction

The mean age at death (MAD) is a summary measure of human longevity with a straightforward interpretation. It gives the average age at which people have died in a given population at a specific point in time. Relying on MAD, however, is not appropriate for most comparative analyses because it is affected by a population's age structure. For instance, MAD is higher for populations with a larger proportion of older than younger individuals, even if the mortality rates are identical in both populations. For this reason, scholars usually prefer period life expectancy at birth ( $e_0$ ), which is age-standardized and widely used for mortality comparisons between countries or periods.

The  $e_0$  indicator is derived from the period life table; i.e., a model in which the life course of a hypothetical population is simulated on the basis of the age-specific mortality rates observed for a given population in one period. The age structure of this hypothetical population, and thus  $e_0$ , results exclusively from the given age-specific mortality rates. Consequently,  $e_0$  is independent of the age structure of the underlying population and reflects the average age at death of the hypothetical period life table population (e.g., Preston, Heuveline, and Guillot 2001; Luy et al. 2020). However, the hypothetical life table population differs substantially from the real population in terms of its lifelong mortality risks. In most countries, the current  $e_0$  value suggests a longer average lifespan than the average number of life years realized by birth cohorts dying today (Guillot and Payne 2019; Shkolnikov et al. 2011; Goldstein and Wachter 2005). This is because mortality levels have been decreasing over time and past mortality rates are usually higher than those currently observed. While the life table population faces only one set of recently observed mortality rates, the birth cohorts dying today have been subjected to

the historical mortality risks of their generation. The current  $e_0$  value can be interpreted as an indicator of potential of longevity because it refers to the average age at death under current mortality rates. However, it should not be used as a benchmark for assessing whether a person's age at death can be considered as comparatively young or old (Guillot and Payne 2019; Vaupel 2002).

A less common way to obtain an age-standardized longevity measure is to assume a population with a constant inflow of annual births, which is closed to migration. The age structure of the constant-birth population is only shaped by cohort-specific mortality rates, which enables us to compare experienced mortality levels between populations without the effects of fertility and migration. The MAD calculated from this population model can be referred to as the standardized mean age at death  $MAD^{std}$  (Sardon 1994; Bongaarts and Feeney 2003). In contrast to  $e_0$ , the measure is based on the mortality trajectories of individual birth cohorts (for all cohorts alive in a given period) and belongs to the family of cross-sectional cohort averages (Luy 2010) such as the cross-sectional average length of life (CAL) (Brouard 1986; Guillot 2003).

The aim of this paper is to compare  $MAD^{std}$  formally and empirically to MAD and  $e_0$ . Mathematically, the three measures can be expressed as weighted averages with different weighting functions. This allows us to apply the formulas provided by Vaupel and Zhang (2012) to study cross-sectional differences in MAD,  $MAD^{std}$ , and  $e_0$ . The differences between these three measures can be attributed to differences in the age distribution of deaths, which is given by the product of age-specific mortality rates and the population's age structure. We find that the size of the gap between the measures depends on the covariance between age and the relative difference between their age distributions of deaths.

We further examine how the three measures react to a sudden increase in age-specific mortality rates; e.g., between the years 2019 and 2020. The formulas presented by Vaupel and Canudas-Romo (2002) on decomposing change in population averages into direct vs. indirect components enable us to express the change in MAD,  $MAD^{std}$ , and  $e_0$  in terms of two covariances: the covariance between age and the change in age-specific mortality rates, and the covariance between age and the change in the age structure.

## 2. Methods

### 2.1 The observed mean age at death (MAD)

The definition of the observed MAD is straightforward. The measure reflects the age at which individuals have died on average in a given period,

$$MAD(t) = \frac{\int_0^{\omega} x \cdot D(x,t) dx}{\int_0^{\omega} D(x,t) dx} \quad (1)$$

where  $D(x, t)$  denotes the observed number of deaths at age  $x$  in time  $t$ . The age-specific number of deaths at a given point in time is the product of people alive at a certain age at time  $t$ ,  $N(x, t)$ , and the age-specific force of mortality at time  $t$ ,  $\mu(x, t)$ , with

$$N(x, t) = B(t - x) \cdot p_c(x, t - x) + M(x, t) \quad (2)$$

and

$$D(x, t) = N(x, t) \cdot \mu(x, t) \quad (3)$$

where  $B(t - x)$  is the number of births  $x$  years before time  $t$  and  $p_c(x, t - x)$  is the probability of individuals born in time  $t - x$  surviving until age  $x$ . The additional term,  $M(x, t)$ , reflects the net number of migrants alive at age  $x$  in time  $t$ . This term can be negative when there are more individuals leaving the population than migrating into the population.

As shown in Equation (3), the  $N(x, t)$  function, i.e., the population's age structure, can be seen as the weighting function for the corresponding age-specific force of mortality. Even if mortality has remained constant between two periods, MAD can increase or decrease through changes in  $N(x, t)$ . Demographers have addressed this issue by using a standard age structure which is held constant (Kitagawa 1964). For instance, the standardized death rate (SDR) uses such a reference population in order to compare mortality levels more appropriately across populations or periods. However, it is important to note that the choice of reference population has a direct impact on the results (Kitagawa 1964; Keyfitz 1985). Therefore, it is more convenient to use  $e_0$  because it does not require a reference population and can only be derived from the  $\mu(x, t)$  function.

## 2.2 Period life expectancy at birth ( $e_0$ )

As described in the introduction,  $e_0$  is derived from a period life table. It is often interpreted as the number of life years that new-borns can expect if the prevailing age-specific mortality rates remain constant in the future. Also,  $e_0$  can be seen as MAD in the stationary population model (Preston, Heuveline, and Guillot 2001) and is calculated from

$$e_0(t) = \frac{\int_0^{\omega} x \cdot d(x,t) dx}{\int_0^{\omega} d(x,t) dx} \quad (4)$$

where  $d(x, t)$  represents the age-specific number of deaths given by the period life table for time  $t$ . Similar to Equation (3), the age distribution of period life table deaths is calculated as the product of the age distribution in the stationary life table population,  $l(x, t)$ , and  $\mu(x, t)$ :

$$l(x, t) = e^{-\int_0^x \mu(a,t) da} \quad (5)$$

$$d(x, t) = l(x, t) \cdot \mu(x, t). \quad (6)$$

In contrast to  $N(x, t)$ , the age distribution in the stationary population is not related to past births, deaths, and migration events but is solely a function of  $\mu(x, t)$ . It refers to a hypothetical scenario, in which recently observed age-specific mortality rates are assumed to remain constant for about 100 years (see, e.g., Preston, Heuveline, and Guillot 2001: 53). This is why  $e_0$  does not require a reference population or, as Heuveline (2023: 6) puts it, the measure can be seen as “internally standardized”.

### 2.3 The standardized mean age at death (MAD<sup>std</sup>)

MAD<sup>std</sup>, an alternative summary measure of longevity, is derived from the constant-birth population. The constant-birth population model does not assume that the age-specific force of mortality is constant over time. Mortality can differ from year to year in accordance with the observed age- and cohort-specific mortality rates (Wilmoth 2005). As described by Guillot (2003), the constant-birth (or standardized) population controls for differences in the number of births by assuming each birth cohort had the same initial size:

$$S(x, t) = B \cdot p_c(x, t - x) \quad (7)$$

where  $B$  is the constant number of annual births. In the case of one birth per year, ( $B = 1$ ), the standardized population is simply given by the  $p_c(x, t - x)$  function, which can be derived from age- and cohort-specific mortality rates:

$$S(x, t) = e^{-\int_0^x \mu_c(a,t-x) da} \quad (8)$$

where  $\mu_c(a, t - x)$  denotes the force of mortality at age  $a$  for the cohort born  $x$  years ago. Previously,  $S(x, t)$  has been discussed in the context of the cross-sectional average length of life (CAL), which is a summary measure of mortality given by integrating  $S(x, t)$  over age (Brouard 1986; Guillot 2003):

$$CAL(t) = \int_0^{\omega} S(x, t). \quad (9)$$

Applying the standardized population to the period force of mortality produces the standardized age distribution of deaths:

$$D^{std}(x, t) = S(x, t) \cdot \mu(x, t). \quad (10)$$

Accordingly, the standardized MAD is defined as

$$MAD^{std}(t) = \frac{\int_0^{\omega} x \cdot D^{std}(x, t) dx}{\int_0^{\omega} D^{std}(x, t) dx}. \quad (11)$$

MAD<sup>std</sup>( $t$ ) can be interpreted as the mean age at death in time  $t$  if there are no fluctuations in the annual number of births and the population is closed to migration (Guillot 2006).

Equations 2 and 7 show formally that both the age distribution of the real population and the standardized age distribution are based on historical mortality data; i.e., mortality risks that cohort members have experienced over the course of their lives. This is why the standardized age distribution is formally closer to the real population than the age structure of the hypothetical period life table population,  $l(x, t)$ . The latter can only correspond to the real population's age structure when mortality does not change over time. Since mortality rates have been decreasing over the last century,  $l(x, t)$  includes a higher proportion of older individuals than the observed and standardized age distribution, leading to a higher average age at death.

Moreover, comparing  $l(x, t)$  and  $S(x, t)$  reveals that both functions reflect the age distribution of a hypothetical population; i.e., the age distribution of the stationary population and the standardized age distribution, respectively, which are solely derived from the force of mortality. Therefore, both  $e_0$  and MAD<sup>std</sup> can be seen as internally standardized measures because they do not use an external reference population. While MAD<sup>std</sup> is formally closer to the observed MAD, the disadvantage of MAD<sup>std</sup> is its high data demand: the empirical calculation of  $S(x, t)$  requires a long time series of detailed cohort-specific data.

It is obvious that differences between the  $N(x, t)$  and  $S(x, t)$  functions are due to the changes in births and migration flows in the real population. Empirically,  $N(x, t)$  will



differ from  $S(x, t)$  because real populations are usually not characterized by constant fertility rates and zero net migration. However, standardizing for changes in births and migration is essential for an unbiased mortality comparison between two populations because it ensures that differences in  $MAD^{std}$  can only result from mortality differences. Table 1 summarizes the three measures compared in this paper with their characteristics regarding the populations on which they are based and the time frame of the mortality rates they summarize.

**Table 1: Summary table of the longevity measures  $MAD(t)$ ,  $MAD^{std}(t)$ , and  $e_0(t)$  with their specific characteristics**

	$MAD(t)$	$MAD^{std}(t)$	$e_0(t)$
Formula	$\frac{\int_0^\omega x \cdot D(x, t) dx}{\int_0^\omega D(x, t) dx}$	$\frac{\int_0^\omega x \cdot D^{std}(x, t) dx}{\int_0^\omega D^{std}(x, t) dx}$	$\frac{\int_0^\omega x \cdot d(x, t) dx}{\int_0^\omega d(x, t) dx}$
Short description	Mean age at death in a real population at time $t$ .	Mean age at death at time $t$ , assuming no fluctuations in the annual number of births and no migration.	Mean age at death in the stationary population at time $t$ .
Weighting function	$D(x, t) = N(x, t) \cdot \mu(x, t)$ Observed number of age-specific deaths in time $t$ .	$D^{std}(x, t) = S(x, t) \cdot \mu(x, t)$ Standardized age distribution of deaths in time $t$ .	$d(x, t) = l(x, t) \cdot \mu(x, t)$ Age distribution of period life table deaths in time $t$ .
Age structure of reference population	$N(x, t)$ Age distribution of the real population in time $t$ .	$S(x, t)$ Age distribution of the constant-birth population in time $t$ .	$l(x, t)$ Age distribution of the stationary population in time $t$ .
Influenced by...			
current mortality	Yes	Yes	Yes
past mortality	Yes	Yes	No
past fertility	Yes	No	No
past migration	Yes	No	No

## 2.4 Examining the difference between two weighted averages

We have defined  $MAD$ ,  $MAD^{std}$ , and  $e_0$  as three different weighted averages,

$$A = \frac{\int_0^\omega x \cdot w(x, t) dx}{\int_0^\omega w(x, t) dx} \tag{12}$$

where the weighting function,  $w(x, t)$ , corresponds to one of the three age distributions of deaths,  $D(x, t)$ ,  $D^{std}(x, t)$ , or  $d(x, t)$ , which are given by  $N(x, t) \cdot \mu(x, t)$ ,  $S(x, t) \cdot \mu(x, t)$ , and  $l(x, t) \cdot \mu(x, t)$ , respectively. To examine differences between these three mortality measures more closely, we use the formulas to calculate the difference between two weighted averages developed by Vaupel and Zhang (2012). Let  $w_1(x, t)$  and  $w_2(x, t)$  be two different weighting functions; e.g., the age distribution of period life table deaths in

time  $t$  or the age distribution of the constant-birth population in time  $t$ . The difference between two averages with alternative weighting functions is then given by

$$A_2(t) - A_1(t) = \frac{cov\left(x, \frac{w_2}{w_1}\right)}{E\left(\frac{w_2}{w_1}\right)} \quad (13)$$

where  $cov\left(x, \frac{w_2}{w_1}\right)$  is the covariance between  $x$  and  $\frac{w_2}{w_1}$ , and  $E\left(\frac{w_2}{w_1}\right)$  denotes the expected (or mean) value of the relative difference between the two alternative weighting functions. Note that the denominator of Equation (13),  $E\left(\frac{w_2}{w_1}\right)$ , is a weighted average with  $w_1$  providing the weights.

Equation (13) reveals that the mean age at death in a population with an older age distribution of deaths will be higher than the mean age at death in a population with a younger age distribution of death. This is because the relative difference of the two weighting functions (the older age distribution of deaths divided by the younger age distribution of deaths) is positively correlated with age.

## 2.5 Examining the change over time of population averages

Previous work by Vaupel and Canudas-Romo (2002) provides helpful equations for decomposing time derivatives of averages. Their work elaborates on the findings of Preston, Himes, and Eggers (1989) and of Schoen and Kim (1992). Using the notation of a dot on top of a variable to denote the derivative of a quantity with respect to time and an acute accent to represent the derivative of the logarithm of a quantity with respect to time, the change of a population average,  $A(t)$ , over time can be expressed as,

$$\dot{A} = cov(x, \dot{w}). \quad (14)$$

In the case of examining the change of an average age at death over time,  $\dot{w}$  is the relative derivative of the given age distribution of deaths; e.g.,  $\dot{D}(x, t)$ ,  $\dot{D}^{std}(x, t)$ , or  $\dot{d}(x, t)$ .

To analyse changes in the average age at death with respect to differences in the age structure, we can substitute  $w(x, t)$  by the product of  $\mu(x, t)$  and the given population age structure,  $k(x, t)$ , yielding  $w(x, t) = \mu(x, t) \cdot k(x, t)$ . Vaupel and Canudas-Romo (2002) have shown that this allows us to express the time derivative of the average age at death in terms of the sum of two covariances:

$$\dot{A} = cov(x, \dot{\mu}) + cov(x, \dot{k}). \quad (15)$$

The first term is the covariance between age and change in age-specific mortality rates, while the second term reflects the covariance between age and change in the age structure of the underlying population. It should be noted that the covariance between two variables reflects compositional effects and cannot be interpreted as direct effects of mortality versus age structure (Vaupel and Canudas-Romo 2002). For instance, when applying Equation (15) to study the dynamics of MAD,  $MAD^{std}$ , and  $e_0$ , all three measures use the same ages  $x$  and the same relative derivative  $\dot{y}$ . Still, the  $cov(x, \dot{\mu})$  terms are not identical for the three measures (see Table 3). This is because the weights in the covariance function for MAD,  $MAD^{std}$ , and  $e_0$  are different.

## 2.6 Data

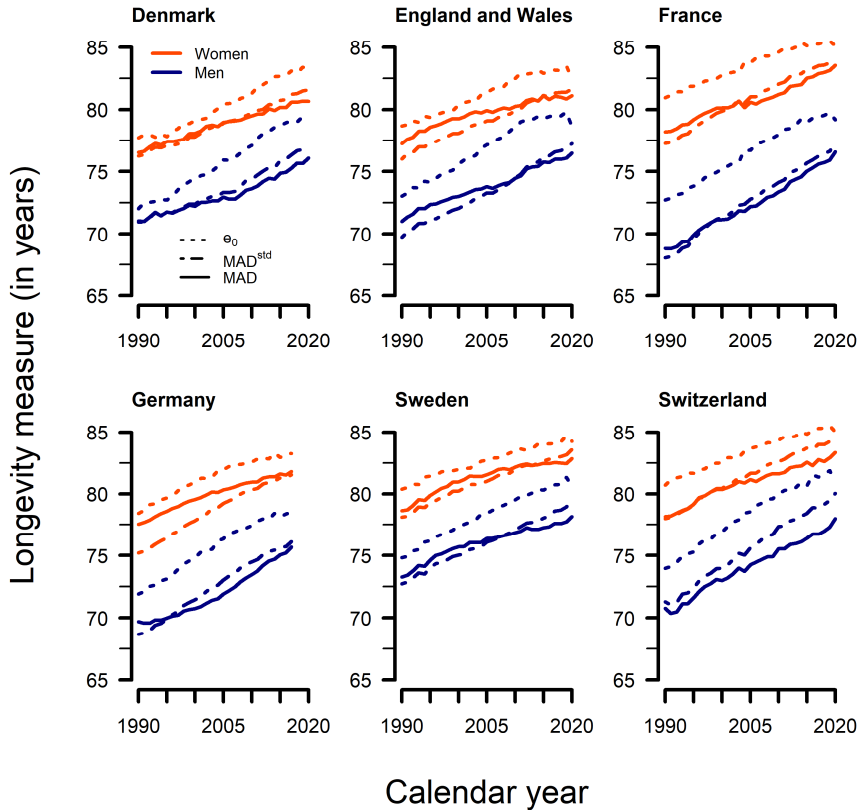
Our empirical results are based on period- and cohort-specific mortality data provided by the Human Mortality Database (HMD 2023) and refer to the time period 1990 to 2020. At the time of the study the required data was available in the HMD for five populations (Denmark, England and Wales, France, Sweden, and Switzerland). In addition, we obtained cohort-specific life tables for Germany from Destatis (2023), which are available up to the year 2017. Consequently, the mortality figures for Germany in our analysis only cover 1990 to 2017.

## 3. Results

### 3.1 Cross-sectional differences in the three longevity measures

Figure 1 presents annual estimates of observed MAD,  $MAD^{std}$ , and  $e_0$  for women and men between 1990 and 2020. The  $e_0$  measure shows the highest values in each year and in every population and gender. The difference between  $e_0$  and the other two measures varies between populations. In Germany and France, for example, the gap between  $e_0$  and the two MAD indicators is greater than in Sweden. Also, the levels of the three mortality measures are more similar among women. This can be attributed to country- and sex-specific differences in historical mortality. Older individuals living in France or Germany today experienced particularly high mortality risks during the Second World War and the post-war era, which is not reflected in the  $l(x, t)$  function. Accordingly, the difference between the observed population and the period life table population is comparatively large in those countries, especially among men.

**Figure 1: Time trends in the observed mean age at death, MAD, standardized mean age at death, MAD<sup>std</sup>, and period life expectancy at birth,  $e_0$ , for women and men in 6 selected populations, 1990–2020**



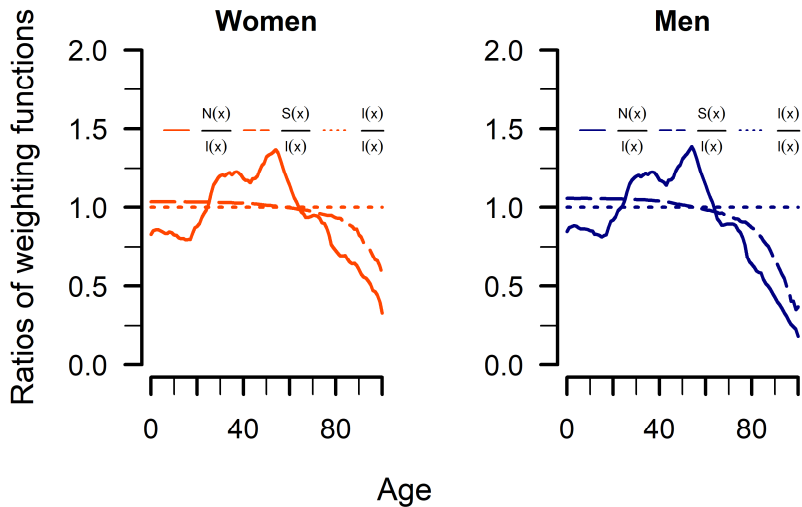
Source: Own calculations based on data from HMD (2023) and Destatis (2023).

Most populations show a crossover of MAD and MAD<sup>std</sup>. In the early 1990s the value for MAD exceeds MAD<sup>std</sup>, but for more recent years MAD<sup>std</sup> indicates a higher average age at death. Differences between MAD and MAD<sup>std</sup> must, by definition, reflect the effects of past fertility and migration. MAD<sup>std</sup> will be greater than MAD when the  $S(x, t)$  function indicates an older age structure than the  $N(x, t)$  function. In this situation, the relative difference between the two weighting functions will be positively correlated with age. This can happen when births have been falling rather than constant over time.

For instance, in 1990 MAD is almost one year higher than  $MAD^{std}$  for French women. In 2019, however,  $MAD^{std}$  exceeds MAD by about 0.6 years (see Table 2). Applying Equation (13) reveals that the covariance between age and  $\frac{D^{std}(x,t)}{D(x,t)}$  has indeed changed between the two points in time (from  $-0.57$  to  $+0.47$ ).

As shown formally above, differences in the three age distributions of deaths stem from differences in the underlying population age structures given by  $N(x, t)$ ,  $S(x, t)$ , and  $l(x, t)$ . A comparison of the three age structures for Swiss women and men in 2019 is depicted in Figure 2. The  $e_0$  indicator provides the highest average ages at death for all analysed populations.

**Figure 2: The relative difference in the age structure of the observed population and the constant-birth population when compared with the age structure of the period life table population, Swiss women and men in 2019**



Source: Own calculations based on data from HMD (2023).

A stagnation or slowing down of improvements in period mortality rates would lead to convergence between  $S(x, t)$  and  $l(x, t)$ , resulting in similar values for  $e_0$  and  $MAD^{std}$ . In the case of observed MAD, the age distribution of the real population is not only shaped by mortality but also includes changes in births and net migration. Past changes in fertility and migration explain the fluctuations in the  $N(x, t)$  function shown in Figure 2. In many European countries the observed MAD might cross over again with  $MAD^{std}$

because of the shift in the age structure when the ‘baby boomers’ reach older ages, leading to a higher relative number of older individuals in the population.

**Table 2: Cross-sectional differences between MAD<sup>std</sup> and the other two longevity measures, MAD and e<sub>0</sub>, for women and men in France and England and Wales, 1990 and 2019**

	France				England and Wales			
	Year 1990		Year 2019		Year 1990		Year 2019	
	Women	Men	Women	Men	Women	Men	Women	Men
MAD <sup>std</sup>	77.27	68.05	83.75	76.64	76.01	69.75	81.39	76.88
MAD	78.21	68.81	83.15	75.92	77.24	70.99	80.85	76.09
Difference (MAD <sup>std</sup> – MAD)	-0.94	-0.76	+0.61	+0.72	-1.23	-1.24	+0.53	+0.80
$cov\left(x, \frac{D^{std}(x, t)}{D(x, t)}\right)$	-0.57	-0.48	+0.47	+0.52	-0.87	-0.85	+0.42	+0.60
$E\left(\frac{D^{std}(x, t)}{D(x, t)}\right)$	+0.61	+0.64	+0.78	+0.72	+0.71	+0.68	+0.78	+0.75
$\frac{cov\left(x, \frac{D^{std}(x, t)}{D(x, t)}\right)}{E\left(\frac{D^{std}(x, t)}{D(x, t)}\right)}$	-0.94	-0.76	+0.61	+0.72	-1.23	-1.24	+0.53	+0.80
e <sub>0</sub>	80.95	72.72	85.52	79.75	78.67	73.04	83.45	79.80
Difference (MAD <sup>std</sup> – e <sub>0</sub> )	-3.67	-4.67	-1.77	-3.11	-2.66	-3.29	-2.06	-2.92
$cov\left(x, \frac{D^{std}(x, t)}{d(x, t)}\right)$	-2.23	-2.97	-1.38	-2.23	-1.89	-2.25	-1.62	-2.20
$E\left(\frac{D^{std}(x, t)}{d(x, t)}\right)$	+0.61	+0.64	+0.78	+0.72	+0.71	+0.68	+0.78	+0.75
$\frac{cov\left(x, \frac{D^{std}(x, t)}{d(x, t)}\right)}{E\left(\frac{D^{std}(x, t)}{d(x, t)}\right)}$	-3.67	-4.67	-1.77	-3.11	-2.66	-3.29	-2.06	-2.92

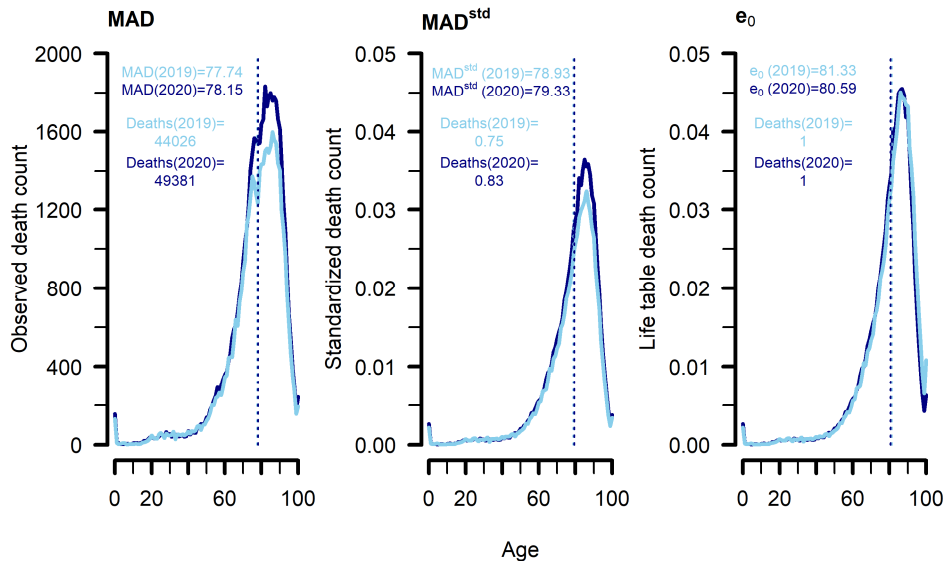
Source: Own calculations based on data from HMD (2023).

### 3.2 MAD, MAD<sup>std</sup>, and e<sub>0</sub> during the mortality change between 2019 and 2020

The time trends in Figure 1 show that e<sub>0</sub> decreased substantially in most analysed populations in the year 2020, while both MADs show further increases. For instance, e<sub>0</sub> decreased by 0.7 years for Swedish men between 2019 and 2020, while both MAD<sup>std</sup> and MAD increased by about 0.4 years over the same period (see Table 3). The changes in the corresponding age-at-death distributions are presented in Figure 3. In addition, Figure 4 depicts the ratio of age-specific mortality rates *m*(*x*) in 2019 and 2020 for Swedish women and men. This graph shows that mortality rates were consistently higher in 2020 for the middle- and old-age groups, whereas mortality rates at younger ages were more

fluctuating. This increase in mortality at older ages resulted in a larger number of deaths for Swedish men in 2020 (49,381 deaths in 2020 vs. 44,026 deaths in 2019). The additional deaths are mostly observed around age 80 and the age distribution of deaths changed primarily in terms of its magnitude. Consequently, observed MAD increased from 77.74 to 78.15 years.

**Figure 3: The age distribution of deaths in the real population, the constant-birth population, and the period life table population for Swedish men in 2019 and 2020**



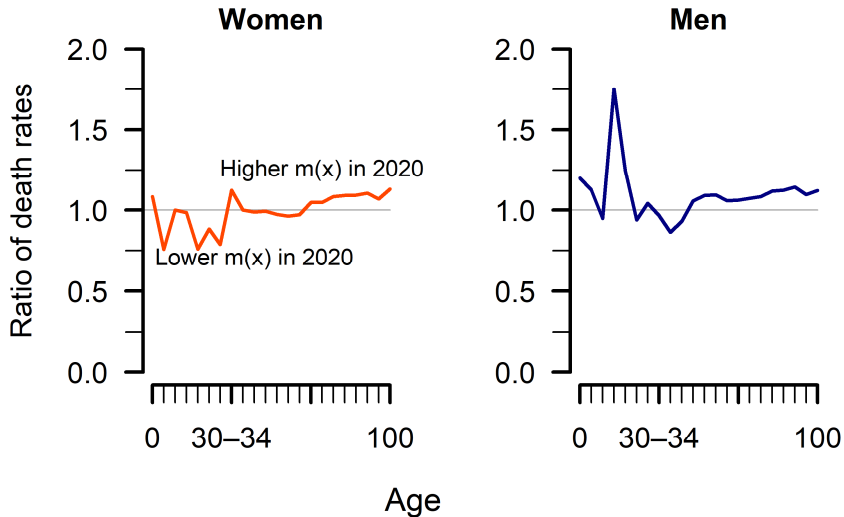
Source: Own calculations based on data from HMD (2023) and Destatis (2023).

The standardized age distribution of deaths controls for differences in the initial size of birth cohorts. For the age distribution of deaths in the constant-birth model shown in Figure 3, we assumed one birth for each birth cohort. The total number of deaths increased from 0.75 to 0.83 between 2019 and 2020, reflecting the increase in age-specific mortality rates. Similar to the observed age distribution of deaths (left panel), the curve changes its magnitude and shifts slightly upward on the age axes. Calculating MAD<sup>std</sup> from the standardized age distributions of deaths in the constant-birth model yields 78.93 and 79.33 years for 2019 and 2020, respectively.

In the period life table, every member of the hypothetical population dies and the total number of life table deaths always equals the life table radix; i.e., the assumed

number of new-borns. In our empirical example we set the radix to 1 to be consistent with MAD<sup>std</sup>.

**Figure 4: Ratios of age-specific mortality rates  $m(x)$  for Swedish women and men between 2019 and 2020**



Source: Data obtained from HMD (2023).

Accordingly, the life table death count does not change between 2019 and 2020. As can be seen in the right panel of Figure 3, the more recent curve shifts slightly to the left, indicating a lower average age at death in 2020 (80.59 years in 2020 and 81.33 years in 2019). The reason for the different impact of the increase in mortality on  $e_0$  as compared to MAD<sup>std</sup> is that the  $S(x, t)$  function is less sensitive than the  $l(x, t)$  function to an increase in period mortality rates from one year to another. As explained above,  $S(x, t)$  includes a large number of historical death rates, whereas  $l(x, t)$  is based on one set of actual age-specific mortality rates only, and thus reacts more strongly to the change in period mortality (Rodríguez 2006; Wilmoth 2005; Guillot 2003).



**Table 3: Observed mean age at death, MAD, standardized mean age at death, MAD<sup>std</sup>, and period life expectancy at birth,  $e_0$ , for women and men in five selected populations, 2019 and 2020**

	Year 2019			Year 2020			Difference 2020-2019		
	MAD	MAD <sup>std</sup>	$e_0$	MAD	MAD <sup>std</sup>	$e_0$	MAD	MAD <sup>std</sup>	$e_0$
<i>Women</i>									
Denmark	80.68	81.49	83.38	80.66	81.57	83.47	-0.02	+0.08	+0.09
England & Wales	80.85	81.39	83.45	81.12	81.64	82.57	+0.27	+0.25	-0.88
France	83.15	83.75	85.52	83.55	84.11	85.11	+0.41	+0.36	-0.41
Sweden	82.44	83.16	84.68	82.84	83.58	84.26	+0.41	+0.42	-0.42
Switzerland	82.92	84.25	85.52	83.36	84.65	85.04	+0.44	+0.40	-0.48
<i>Men</i>									
Denmark	75.65	76.74	79.43	76.07	77.11	79.58	+0.42	+0.37	+0.14
England & Wales	76.09	76.88	79.80	76.48	77.24	78.60	+0.39	+0.36	-1.21
France	75.92	76.64	79.75	76.57	77.30	79.19	+0.65	+0.66	-0.57
Sweden	77.74	78.93	81.33	78.15	79.33	80.59	+0.42	+0.40	-0.74
Switzerland	77.30	79.47	81.87	78.00	80.05	80.99	+0.70	+0.58	-0.88

Source: Own calculations based on data from HMD (2023) and Destatis (2023).

The decomposition of the change in the three mortality measures between 2019 and 2020 reveals that the decrease in  $e_0$  can be attributed to the negative covariance (or correlation) between age and the change in age structure of the life table population. The correlation between age and the relative time derivatives of  $N(x, t)$ ,  $S(x, t)$ , and  $l(x, t)$  are shown in Figure 5, while the values for the covariance are presented in Table 4. Note that we used the approximation of derivatives as suggested by Vaupel and Canudas-Romo (2002: 12).

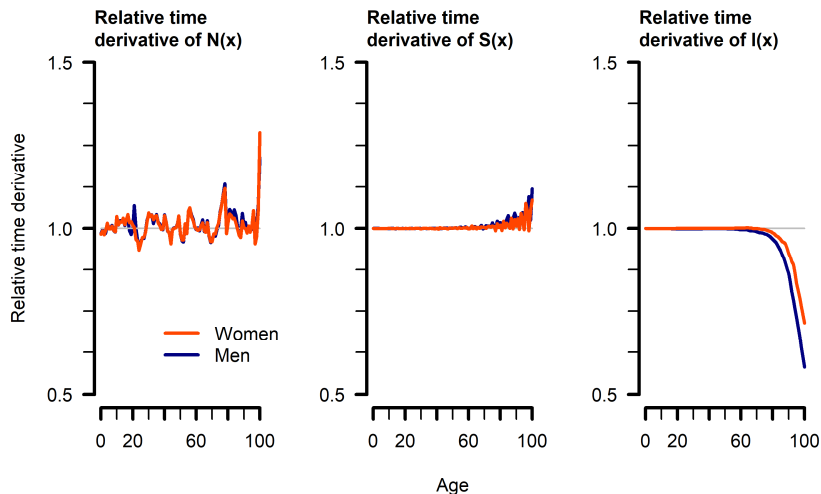
In our case, it is difficult to separate the change in  $e_0$  or MAD<sup>std</sup> into the effects of changes in mortality rates vs. population structure because  $l(x, t)$  and  $S(x, t)$  are actually functions of age-specific mortality rates themselves. Still, the exercise can help us to understand why MAD and MAD<sup>std</sup> increase in 2020, whereas  $e_0$  decreases: both the observed age structure and the standardized age distribution have shifted to older ages, as indicated by the positive covariance between  $\hat{k}$  and age. This results in an increase in the average age at death.

**Table 4: Decomposition of the change in the mean age at death, MAD, standardized mean age at death, MAD<sup>std</sup>, and period life expectancy at birth, e<sub>0</sub>, for women and men in Sweden and Switzerland between 2019 and 2020**

	MAD	Women MAD <sup>std</sup>	e <sub>0</sub>	MAD	Men MAD <sup>std</sup>	e <sub>0</sub>
<i>Sweden</i>						
A(2019)	82.44	83.16	84.68	77.74	78.93	81.33
A(2020)	82.84	83.58	84.26	78.15	79.33	80.59
$\dot{A}(2019.5)$	+0.41	+0.42	-0.42	+0.42	+0.40	-0.74
$Cov(x, \dot{\mu})$	+0.35	+0.31	+0.27	+0.28	+0.26	+0.23
$Cov(x, \dot{k})$	+0.05	+0.11	-0.69	+0.13	+0.14	-0.97
$Cov(x, \dot{\mu}) + Cov(x, \dot{k})$	+0.40	+0.42	-0.42	+0.42	+0.40	-0.74
<i>Switzerland</i>						
A(2019)	82.92	84.25	85.52	77.30	79.47	81.87
A(2020)	83.36	84.65	85.04	78.00	80.05	80.99
$\dot{A}(2019.5)$	+0.44	+0.40	-0.48	+0.70	+0.58	-0.88
$Cov(x, \dot{\mu})$	+0.33	+0.26	+0.23	+0.48	+0.41	+0.37
$Cov(x, \dot{k})$	+0.11	+0.13	-0.71	+0.22	+0.18	-1.25
$Cov(x, \dot{\mu}) + Cov(x, \dot{k})$	+0.43	+0.39	-0.48	+0.70	+0.58	-0.88

Source: Own calculations based on data from HMD (2023).

**Figure 5: Change in age structures  $N(x)$ ,  $S(x)$ , and  $l(x)$  for Swedish women and men between 2019 and 2020**



Source: Own calculations based on data from HMD (2023).

By contrast, in the life table the increase in period mortality rates in 2020 makes the age structure younger. As a consequence, the covariance between  $\hat{k}$  and age is negative and  $e_0$  decreases between the two points in time. The covariance between  $\hat{y}$  and age is consistently positive, and thus less important for explaining the increase vs. decrease in the three measures between 2019 and 2020. The reason why the magnitude of this covariance differs between the three measures – even though they all use the same ages  $x$  and age-specific mortality rates  $\mu(x, t)$  – is the different weighting in the covariance function (see Equation A2 in the Appendix).

## 4. Discussion

In this paper we have defined MAD,  $MAD^{std}$ , and  $e_0$  as weighted averages with three different weighting functions: the observed death distribution, the standardized death distribution, and the life table death distribution. The work of Vaupel and Canudas-Romo (2002) and Vaupel and Zhang (2012) provided the formal basis for relating cross-sectional differences and time changes to differences in the underlying age structures of the three measures.

Our results reveal that  $e_0$  consistently exceeds MAD and  $MAD^{std}$  due to higher proportions of older individuals in the hypothetical life table population as compared to the real population and the hypothetical constant-birth population.  $MAD^{std}$  is less than  $e_0$  as long as mortality rates decrease over time. When mortality rates remain unchanged for a long time (about 100 years)  $MAD^{std}$  will equal  $e_0$  (Bongaarts and Feeney 2003). Consequently, the size of the gap between  $MAD^{std}$  and  $e_0$  depends on the difference between the mortality schedule implied by period mortality rates and the past mortality experience of cohorts. In Sweden and Switzerland,  $MAD^{std}$  almost reached the level of  $e_0$  in the year 2020 among women. This is because both populations show comparatively low historical mortality rates, which makes the difference between the  $S(x, t)$  and  $l(x, t)$  functions smaller than in other populations. The gap between the two functions became even smaller when period mortality rates increased in 2020. Trends in MAD are less related to mortality changes. When we look at levels and trends in the  $N(x, t)$  function, the age structure is mostly driven by fertility changes. Consequently, the proportion of deaths is related to the size of the cohorts, with particularly strong effects from the baby boomer generation. Therefore, MAD is lower than  $MAD^{std}$  and  $e_0$  because MAD involves higher weights on mortality rates in middle ages. As soon as the baby boomer generation reaches older ages, their high weight shifts the age structure to older ages and MAD increases markedly.

To make appropriate mortality comparisons between populations and period, longevity measures should be standardized for the effects of fertility and migration. We

have shown that MAD<sup>std</sup> and  $e_0$  can be seen as standardized in this respect, and therefore they provide valuable tools for comparing mortality levels between populations or across time. It might also be preferable that a longevity measure suggests an average age at death that can serve as an orientation or benchmark age for individuals living and dying today. A person reaching his/her 80th birthday might be interested in the average lifespan of his/her peers. An approach that provides such insights is lagged cohort life expectancy (LCLE) (Guillot and Payne 2019; Guillot and Kim 2011). Guillot and Payne (2019: 406) describe LCLE as follows: “Instead of plotting cohort life expectancy against the cohort’s year of birth,  $c$ , as typically done, cohort life expectancy is lagged by its own value and plotted against the cohort’s mean year at death,  $t(c)$ ”. For instance, LCLE for French women in 2000 is about 72 years, indicating that the cohort born 72 years ago (in 1928) reaches its average age at death in the year 2000. Because LCLE provides a reference age that separates early deaths from late deaths, scholars have compared LCLE to other longevity measures such as  $e_0$  and CAL (see Guillot and Kim 2011). The current  $e_0$  level suggests a substantially later average age at death than LCLE, as it reflects only recently observed mortality rates ( $e_0$  is 82.8 years for French women in 2000). Even though CAL and LCLE are conceptually close to each other, they are still different longevity measures (CAL is 76.66 years). The gap between MAD<sup>std</sup> and LCLE is even larger than CAL vs. LCLE because the MAD<sup>std</sup> value is usually higher than CAL and hence closer to  $e_0$  (MAD<sup>std</sup> is 79.88 years). A table with empirical values for MAD<sup>std</sup>, CAL, LCLE, and  $e_0$  can be found in the Appendix.

Finally, we have analysed how the three mortality measures analysed in this paper reflect a sudden increase in period mortality rates. Our results are in line with previous findings, showing that only  $e_0$  decreases in the year with higher period mortality rates (Wilmoth 2005; Rodriguez 2006). However, what is often overlooked is that the magnitude of the change in  $e_0$  depends not only on the magnitude of changes in age-specific mortality rates between two periods but also on the initial level of  $e_0$ . The initial level of  $e_0$  and the corresponding  $l(x, t)$  function provide weights for changes in age-specific mortality rates (Pollard 1982; Arriaga 1984; Vaupel 1986; Vaupel and Canudas-Romo 2003; Goldstein and Lee 2020; Modig, Rau, and Ahlbom 2020). In other words, a population with a higher  $e_0$  is affected differently by an identical change in period mortality rates between two periods than a population with a lower  $e_0$  level. This can have concrete consequences for life expectancy in situations like the recent Covid-19 pandemic. If we assume, for example, that Swedish men would have experienced their  $e_0$  level of 1989 in the year 2019 (74.78 years) and the population had experienced exactly the same change in period mortality rates between 2019 and 2020 (measured through the ratios of mortality rates depicted in Figure 4), the decrease in  $e_0$  would have been 0.9 years instead of the actual 0.7 years.

To avoid the effects of weights on measuring mortality differentials, it has also been suggested that the geometric mean of age-specific mortality rates be used (Schoen 1970, 1976). The index called del ( $\nabla$ ) summarizes age-specific mortality rates but gives equal weight to each age group. We used  $\nabla$  to quantify the change in mortality rates shown in Figure 4 and found that Swedish women and men experienced similar relative differences in age-specific mortality rates between 2019 and 2020. The comparison of  $e_0$  and  $\nabla$  shows that populations ranked in terms of their  $e_0$  losses do not necessarily reflect which population experienced the highest or lowest changes in age-specific mortality rates (see Appendix). However, results for the four selected countries indicate rather small differences between  $\nabla$  and  $e_0$ ; i.e.,  $e_0$  and  $\nabla$  reflect the increase in period mortality rates observed between 2019 and 2020 similarly.

## 5. Conclusions

The constant-birth population model provides an alternative way to obtain a standardized mortality indicator, referred to as  $MAD^{std}$ . The measure is closely related to CAL. While CAL is not an average age at death but instead refers to the relative size of the constant-birth population,  $MAD^{std}$  is the average age at death in the constant-birth population. Hence it makes sense to focus on  $MAD^{std}$  instead of CAL because it allows a comparison with other average ages at death, such as the observed MAD and  $e_0$ .

Given that the observed MAD value is affected by changes in births and migration flows, our comparison has focused mainly on differences between the two standardized mortality measures,  $MAD^{std}$  and  $e_0$ . Ultimately, the difference can be attributed to the underlying weighting functions,  $S(x, t)$  vs.  $l(x, t)$ . The former depends on the past mortality experience of cohorts which are present in the population in a certain period. For instance, individuals living in Sweden or Switzerland were subjected to lower mortality rates over the course of their lives than people in Germany or England and Wales.  $MAD^{std}$  reflects the differences in historical mortality, making it valuable for evaluating a population's mortality level in terms of its lifelong mortality risks.

Yet  $MAD^{std}$  and  $e_0$  are based on hypothetical population models, making it difficult to infer from them the average age at death for any real birth cohort. In fact, neither measure corresponds to LCLE.  $MAD^{std}$  and  $e_0$  are usually higher than the average age at death for birth cohorts dying today. In contrast to  $e_0$ ,  $MAD^{std}$  is not suitable for detecting the most recent changes in period mortality and is therefore less useful for any public health authority that needs to implement health measures in a timely fashion. In sum,  $MAD^{std}$  provides an alternative way to obtain a standardized mortality measure, but cannot be seen as a substitute for  $e_0$  because the two measures reflect mortality experiences that are based on different time frames.

## **6. Acknowledgments**

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## Appendix

### Derivation of the lagged cohort life expectancy (LCLE)

Let  $e_0^c(t)$  be the life expectancy at birth for the cohort born in year  $t$ . The average year at death for this cohort is  $c = t + e_0^c(t)$ . In the LCLE approach, cohort life expectancy is not plotted against its year of birth  $t$ , but lagged by its own value and plotted against the cohort's mean year at death,  $c$  (Guillot and Payne 2019). It is possible to estimate LCLE for recent years but it requires making assumptions about future mortality rates; i.e., forecasting age-specific mortality rates in order to estimate  $e_0^c(t)$  for more recent birth cohorts. For this paper, it is sufficient to compare LCLE to other longevity measures for years with available data. For instance, the youngest Danish cohort with available  $e_0^c$  in the HMD was born in 1931. Women of this cohort lived on average 71.18 years, men lived 66.66 years. Hence the mean year at death for women is 2002.18 (1931 + 71.18) and for men 1997.66 (1931 + 66.66). These are the most recent LCLE values we can provide for Denmark based on observed data.

**Table A-1: Period life expectancy at birth  $e_0$ , cross-sectional average length of life CAL, standardized mean age at death MAD<sup>std</sup>, and lagged cohort life expectancy LCLE, women, various countries**

	Year	$e_0$	CAL	MAD <sup>std</sup>	LCLE	Exact year $c$
Denmark	1998	78.87	75.59	77.37	71.18	1998.18
	1999	78.87	75.74	77.78	71.41	1999.41
	2000	79.11	75.92	77.80		
England and Wales	2001	79.20	76.10	78.11	71.59	2000.59
	1998	79.86	75.55	77.84	71.31	1998.31
	1999	79.92	75.80	78.07		
France	2000	80.35	76.06	78.09	71.56	1999.56
	2001	80.57	76.33	78.30	72.29	2001.29
	2000	82.76	76.66	79.88	71.74	1999.74
Sweden	2001	82.90	77.02	80.03		
	2002	83.00	77.34	80.32	72.55	2001.55
	2003	82.92	77.66	80.77		
	2004	83.81	77.98	80.51	73.61	2003.61
Switzerland	1999	81.87	78.45	80.21	74.61	1998.61
	2000	81.99	78.67	80.23	74.85	1999.85
	2001	82.03	78.90	80.47	74.85	2000.85
Switzerland	2002	82.07	79.11	80.76	75.33	2002.33
	2005	83.67	79.95	81.67	76.63	2004.63
	2006	83.81	80.22	81.69	77.13	2006.13
	2007	83.93	80.48	81.98	77.44	2007.44
	2008	84.14	80.73	82.32		
	2009	84.16	80.96	82.55	77.78	2008.78

Source: Own calculations based on data from HMD (2023).

**Table A-2: Period life expectancy at birth  $e_0$ , cross-sectional average length of life CAL, standardized mean age at death MAD<sup>std</sup>, and lagged cohort life expectancy LCLE, men, various countries**

	Year	$e_0$	CAL	MAD <sup>std</sup>	LCLE	Exact year $c$
Denmark	1991	72.47	68.91	70.98	64.63	1990.63
	1992	72.56	69.08	71.35	65.28	1992.28
	1993	72.60	69.26	71.71	65.40	1993.40
	1994	72.78	69.41	71.46		
	1995	72.73	69.58	71.81	65.51	1994.51
	1996	73.05	69.72	71.71	65.62	1995.62
England and Wales	1992	73.73	68.25	70.42	64.51	1991.51
	1993	73.68	68.56	70.84	65.27	1993.27
	1994	74.21	68.88	70.85		
	1995	74.14	69.20	71.15	66.31	1995.31
France	1990	72.72	65.62	68.05	62.18	1990.18
	1991	72.87	65.99	68.14		
	1992	73.14	66.35	68.41	63.21	1992.21
	1993	73.25	66.71	68.84		
Sweden	1994	73.64	67.07	69.01	64.16	1994.16
	1990	74.81	70.65	72.71	67.35	1990.35
	1991	74.94	70.88	72.85		
	1992	75.36	71.12	73.21	67.87	1991.87
	1993	75.49	71.37	73.60	68.06	1993.06
	1994	76.08	71.62	73.51	68.33	1994.33
	1995	76.17	71.89	74.01		
	1996	76.52	72.14	74.34	68.53	1995.53
	1997	76.70	72.40	74.43	69.05	1997.05
	1998	76.86	72.67	74.65		
	1999	77.07	72.93	74.91	69.51	1998.51
	1999	77.07	72.93	74.91	69.46	1999.46
Switzerland	2000	77.38	73.18	75.03		
	2001	77.54	73.46	75.12	70.38	2001.38
	1991	74.11	69.82	71.06	67.13	1991.13
	1992	74.45	70.09	71.26		
	1993	74.90	70.37	71.92	67.89	1992.89
	1994	75.14	70.65	71.99	67.93	1993.93
	1995	75.29	70.92	72.43		
	1996	75.93	71.20	72.95	68.72	1995.72
	1997	76.24	71.48	73.33	69.05	1997.05
	1998	76.31	71.77	73.57		
	1999	76.78	72.05	73.94	69.66	1998.66
	2000	76.91	72.33	73.95	69.96	1999.96
	2001	77.35	72.64	74.26		
2002	77.72	72.94	74.64	70.77	2001.77	

Source: Own calculations based on data from HMD (2023).

### Derivation of the geometric mean of age-specific mortality rates ( $\nabla$ )

The del index ( $\nabla$ ) was introduced by Schoen (1970) and is defined as

$$\nabla(t) = \left(\prod_{i=1}^n m(x, t)\right)^{1/n}, \quad (\text{A1})$$

where  $m(x, t)$  are the age-specific mortality rates in time  $t$ . Both  $e_0$  and  $\nabla$  are functions of  $m(x, t)$ . However, the difference is that  $\nabla$  gives equal weight to each age. Following Schoen (1970), we compare the ratio of  $\nabla$  to the ratio of life table crude death rate ( $1/e_0$ ), instead of focusing on absolute losses in  $e_0$  (Table A-3).

One could make the argument that age-specific mortality rates at younger ages with lower numbers of deaths are highly fluctuating over time in low-mortality countries with smaller population sizes, such as Switzerland. In addition, COVID-19 has particularly affected mortality rates at older ages. For this reason, we provide the same analysis, considering only mortality at age 50 and older (Table A-4). We used mortality rates for 5-year intervals, which were obtained from the HMD (2023).

**Table A-3: Change in the geometric mean of age-specific mortality rates  $\nabla$  and in the life table crude death rate  $1/e_0$  between 2019 and 2020**

	Geometric Mean ( $\nabla$ )			Life Table Crude Death Rate ( $1/e_0$ )		
	2019	2020	Ratio	2019	2020	Ratio
<i>Women</i>						
Denmark	0.0029	0.0027	0.9231	0.0120	0.0120	0.9989
France	0.0028	0.0028	0.9938	0.0117	0.0117	1.0048
England and Wales	0.0033	0.0033	1.0157	0.0120	0.0121	1.0107
Switzerland	0.0024	0.0026	1.0744	0.0117	0.0118	1.0056
Sweden	0.0028	0.0028	0.9906	0.0118	0.0119	1.0050
<i>Men</i>						
Denmark	0.0045	0.0046	1.0235	0.0126	0.0126	0.9982
France	0.0050	0.0051	1.0118	0.0125	0.0126	1.0072
England and Wales	0.0048	0.0051	1.0635	0.0125	0.0127	1.0153
Switzerland	0.0038	0.0042	1.1009	0.0122	0.0123	1.0109
Sweden	0.0040	0.0043	1.0903	0.0123	0.0124	1.0092

Source: Own calculations based on data from HMD (2023).

**Table A-4: Change in the geometric mean of age-specific mortality rates  $\nabla$  and in the life table crude death rate  $1/e_0$  between 2019 and 2020, considering only mortality at ages 50 and older**

	Geometric Mean ( $\nabla$ )			Life Table Crude Death Rate ( $1/e_0$ )		
	2019	2020	Ratio	2019	2020	Ratio
<i>Women</i>						
Denmark	0.0315	0.0309	0.9803	0.0118	0.0118	0.9985
France	0.0250	0.0263	1.0519	0.0115	0.0116	1.0056
England and Wales	0.0300	0.0332	1.1072	0.0118	0.0119	1.0107
Switzerland	0.0246	0.0258	1.0454	0.0116	0.0116	1.0052
Sweden	0.0268	0.0281	1.0482	0.0117	0.0117	1.0056
<i>Men</i>						
Denmark	0.0455	0.0448	0.9849	0.0123	0.0123	0.9977
France	0.0420	0.0450	1.0709	0.0122	0.0123	1.0081
England and Wales	0.0412	0.0466	1.1324	0.0122	0.0124	1.0148
Switzerland	0.0358	0.0393	1.1002	0.0120	0.0121	1.0096
Sweden	0.0372	0.0407	1.0940	0.0121	0.0122	1.0089

Source: Own calculations based on data from HMD (2023).

### Approximation of derivatives in discrete time

We follow Vaupel and Canudas-Romo (2002: 12) and use the mid-point approximation for finding the derivative of function  $v(x, t)$ . First, we estimate the mid-point value of the function  $v(x, t)$ ,

$$v\left(x, t + \frac{h}{2}\right) \approx [v(x, t) \cdot v(x, t + h)]^{\frac{1}{2}}, \quad (\text{A2})$$

where  $h$  denotes the number of years in the analysed time interval. Then the relative derivative for the function  $v(x, t)$  is given by,

$$\dot{v}\left(x, t + \frac{h}{2}\right) \approx \frac{\ln\left[\frac{v(x, t+h)}{v(x, t)}\right]}{h}. \quad (\text{A3})$$

Finally, the derivative of the function  $v(x, t)$  is estimated by,

$$\dot{v}\left(x, t + \frac{h}{2}\right) = \dot{v}\left(x, t + \frac{h}{2}\right) \cdot v\left(x, t + \frac{h}{2}\right). \quad (\text{A4})$$

