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Research Article

Tempo effects in period TFR: Inspecting the role of shape and scale variations in a cohort model

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Abstract

BACKGROUND
The total fertility rate (TFR) is a fundamental demographic measure widely used for assessing fertility trends in populations. However, the TFR is susceptible to distortion due to timing effects, which can confound the understanding of true fertility patterns.

OBJECTIVE
This study investigates the impact of changes in the distribution of fertility rates on the period total fertility rate (PTFR) from a cohort perspective.

METHODS
We adopt a model representation that separates the quantum (the fertility that would occur without timing changes) from the tempo (timing changes) components. Using a skewed normal distribution to fit cohort fertility schedules, we explore the impact of variations in cohort mean age at childbearing, variance, and skewness on the PTFR. Simulation studies are also conducted to investigate the transient behavior of the TFR.

RESULTS
We demonstrate that the tempo distortion in PTFR depends on the speed and magnitude of shifts in scale and shape parameters. Adjusting PTFR for these variations yields different results compared to adjustments based solely on mean shifts, highlighting the importance of considering all tempo parameters.

CONCLUSIONS
Analyzing tempo fluctuations from a cohort perspective reveals their significant impact on PTFR estimates. Additionally, it becomes evident that the changes observed at the cohort level are predominantly reflected in the period shift of the mean age at childbearing.

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CONTRIBUTION
This research contributes to the ongoing discussion regarding the impact of cohort fertility schedule changes on PTFR. Our cohort-focused approach sheds light on the role of scale and shape variations and their implications for understanding fertility trends, emphasizing the need for a comprehensive assessment of tempo distortions in demographic analyses. The findings bear significance for policy evaluation in the context of demographic changes.

1. Introduction

The period total fertility rate (PTFR) measure is widely known to be affected by tempo-effect distortion, and there is a wide literature investigating ways to identify the so-called quantum and tempo components. The quantum component can be defined as the total fertility rate (TFR) that would have been observed in the absence of changes in the timing of childbearing during the period in which the TFR is measured, and might be the real object of analysis. The tempo component equals the distortion that occurs due to timing changes. In this perspective, several methods to adjust the TFR have been elaborated, and the most prominent has been proposed by Bongaarts and Feeney (1998) (BF method). They show that when the level of fertility does not change (quantum), but the mean age at childbearing increases through a constant shift in the period fertility schedule (tempo), the TFR is distorted, but it can be adjusted by a factor that takes into account the annual increase in the mean age of childbearing. Their adjustment formula is based on the assumption that women of all ages who have children of a given birth order in a year postpone births to the same extent regardless of their age or cohort.

Some criticism (van Imhoff and Keilman 2000; Kim and Schoen 2000; van Imhoff 2001) to the BF method have been raised: Kim and Schoen (2000) insist that the mathematical basis of adjustment holds only under very restrictive conditions, and that with those constraints even slightly relaxed, the adjusted TFR is quite volatile in the presence of modest fertility fluctuations. Van Imhoff and Keilman (2000) argue that Bongaarts and Feeney adjustment procedure is based on fertility measures (i.e., the parity-specific fertility rates) unsuitable for tempo adjustment because the use of frequencies exaggerates the effect of tempo distortions, and the assumption underlying their method (i.e., period-by-period timing changes are independent of age and cohort) is not supported by the data. Another criticism is that the BF method accounts only for mean variations, commonly known as temporal shift, but not for shape or scale changes in birth distribution (Kohler and Philipov 2001; van Imhoff 2001). The BF method however has the advantage of providing a tempo adjustment based on only period data by assuming that all cohorts involved postpone their fertility the same way, thus adjustment can be made for recent years.

In this paper, we focus on the criticism moved by Kohler and Philipov (2001) and van Imhoff (2001) and investigate whether shape and scale variation in the fertility schedule also determine a tempo distortion. Kohler and Philipov (2001) test if variance changes in fertility schedules also affect PTFR, finding that a relevant distortion occurs only if the change is not constant over time. Similarly, Yi and Land (2001) find that while the BF method is generally robust, it may be sensitive to large and time-varying changes in the shape of fertility schedules. Both of the abovementioned studies take a period perspective (i.e. tempo and shape) and are measured on period fertility rates. Here we take a cohort perspective, as Goldstein and Cassidy (2014) do. The debate on whether period or cohort effects are mostly relevant in determining fertility trends variations is an old one, but we agree with Bongaarts and Sobotka (2012), who state that with any change in fertility at a given age, cohort and period can be described both with a period and a cohort perspective.

To identify how changes in the distribution of cohort fertility rates are reflected in the PTFR, the model representation proposed by Hoem et al. (1981), which separates the quantum from timing changes, was used. Timing fertility schedules are estimated by employing the skew-normal distribution (Mazzuco and Scarpa 2015), the parameters of which can be interpreted in terms of mean, variance, and skewness index. We will consider several cases in which the cohort schedule changes in terms of location (i.e., mean age at birth), scale (i.e., concentration of fertility around the mean), and shape (skewness of fertility schedule), but the cohort TFR (CTFR) remains fixed and will find out how the PTFR changes accordingly. We will also analyze how a change on cohort timing parameters (mean, variance, and skewness) affects the timing of period data.

The manuscript is organized as follows: In Section 2 the model is described and then applied to the data presented in Section 3; in Section 4, we explain the results of several simulations and show the relationships between cohort timing parameters change and PTFR. In Section 5 we apply the method to Swedish time series to understand the real impact of cohort changes on the calculation of the period total fertility rate, while in Section 6 we conclude.

2. Method

Our starting point is considering the fertility model representation proposed by Hoem et al. (1981), which may be written as

$$g(x; R, \theta_2, \ldots, \theta_r) = R \cdot h(x; \theta_2, \ldots, \theta_r), \tag{1}$$

where $h(\cdot;\theta_2,\ldots,\theta_r)$ is a probability density function on the real line with $r-1$ parameters, and R is the r-th parameter representing the TFR. Note that Model (1) separates the quantum (given by parameter R) and tempo components – the function $h(\cdot)$. Therefore, it can be profitably used to determine the effect of changing the timing of fertility schedules on the quantum parameter. Here, we consider the proposal by Mazzuco and Scarpa (2015), which suggests using a skew-normal distribution to represent the fertility schedule $h(\cdot)$, although the proposed method can be applied to any distribution that provides a satisfactory fit to the data and for which the first three moments exist and can be explicitly calculated. Examples include the Hadwiger distribution (Chandola, Coleman, and Hiorns 1999) or the distribution underlying the model proposed by Peristera and Kostaki (2007). The skew-normal distribution has the advantage that its parameters can be easily represented in terms of location, scale, and shape parameters. Given Model (1), let us assume that the probability density function $h(\cdot;\theta_2,\ldots,\theta_r)$ follows a skew-normal distribution,

$$h(x|\omega,\xi,\alpha) = 2\omega^{-1}\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left\{\alpha\left(\frac{x-\xi}{\omega}\right)\right\}, \tag{2}$$

where ϕ and Φ are the probability density function and the cumulative distribution function of the standard normal distribution, respectively. This distribution was introduced by Azzalini (1985) and further studied by Azzalini and Capitanio (2003). Azzalini and Capitanio (2003) propose a useful reparametrization to overcome the issue of the singularity of matrix information when $\alpha = 0$, which also allows rewriting Equation (2) in terms of mean (μ), variance (σ^2), and skewness index (γ) (ξ is the location parameter, but it is not the mean, ω is not the variance, and α is not the symmetry of the distribution):

$$\begin{aligned}
\mu &= \xi + b\omega\frac{\alpha}{\sqrt{1+\alpha^2}} = \xi + \omega\delta(\alpha);\\
\sigma &= \omega\left(1-b^2\delta(\alpha)^2\right)^{\frac{1}{2}};\\
\gamma &= \frac{4-\pi}{2}\frac{b^3\alpha^3}{\left[1+(1-b^2)\alpha^2\right]^{\frac{3}{2}}};
\end{aligned} \tag{3}$$

where $b = \sqrt{2/\pi}$.

Let us suppose that the time schedule from cohort t_1 to cohort t_2 changes, so that the schedule in t_1 is $h(x|\mu_1, \sigma_1, \gamma_1)$ and the schedule in t_2 is $h(x|\mu_2, \sigma_2, \gamma_2)$. Therefore, the schedule in t_2 can be written as

$$\begin{aligned} h(x|\mu_2, \sigma_2, \gamma_2) &= h(x|\mu_2, \sigma_2, \gamma_2) \cdot \frac{h(x|\mu_1, \sigma_1, \gamma_1)}{h(x|\mu_1, \sigma_1, \gamma_1)} = \\ &= h(x|\mu_1, \sigma_1, \gamma_1) \cdot \lambda(x, \mu_1, \mu_2, \sigma_1, \sigma_2, \gamma_1, \gamma_2), \end{aligned} \quad (4)$$

where $\lambda(x, \mu_1, \mu_2, \sigma_1, \sigma_2, \gamma_1, \gamma_2)$ is the ratio between $h(x|\mu_2, \sigma_2, \gamma_2)$ and $h(x|\mu_1, \sigma_1, \gamma_1)$. Thus, if mean, variance, and skewness parameters vary and the quantum parameter R remains fixed, the age-specific fertility rate can be written as[3]

$$\begin{aligned} f_x^2 &= R \cdot h(x|\mu^2, \sigma^2, \gamma^2) = R \cdot h(x|\mu^1, \sigma^1, \gamma^1)\lambda^1(x) \\ &= f_x^{t_0} \lambda_2^1(x). \end{aligned} \quad (5)$$

This means that we can adjust the age-specific fertility rate f_x^2, getting the value that would have been observed if the timing of fertility schedule had remained stable:

$$\tilde{f}_x^2 = \frac{f_x^2}{\lambda_2^1(x)}. \quad (6)$$

More generally, if the quantum parameter R remains fixed between t_0 and t, while the tempo parameters vary, we have

$$\begin{aligned} f_x^t &= R \cdot h(x|\mu^t, \sigma^t, \gamma^t) = R^{t_0} \cdot h(x|\mu^{t_0}, \sigma^{t_0}, \gamma^{t_0})\lambda^{t_0}(x) \\ &= f_x^{t_0} \lambda_t^{t_0}(x), \end{aligned} \quad (7)$$

and the age-specific fertility rate f_x^t can be adjusted by

$$\tilde{f}_x^t = \frac{f_x^t}{\lambda^{t_0}(x)} = R \cdot h(x|\mu^{t_0}, \sigma^{t_0}, \gamma^{t_0}). \quad (8)$$

Note that the integral of $\tilde{f}_x^t$ but the moments of birth distribution (mean, variance, and skewness) are set to values in t_0. We now consider a scenario similar to that proposed by Bongaarts and Feeney (1998): We assume that the CTFR is constant over cohorts

[3] We simplify the notation by imposing $\lambda_2^1(x) = \lambda(x, \mu^1, \mu^2, \sigma^1, \sigma^2, \gamma^1, \gamma^2)$.

but the timing changes. Unlike Bongaarts and Feeney, here we assume that cohort timing changes, not period one, as suggested by Goldstein and Cassidy (2014). Moreover, we neither specify the functional form with which fertility timing varies nor limit the mean changes, but we do allow σ and γ to vary. Following Model (1), and considering $h(x; \mu(\tau), \sigma(\tau), \gamma(\tau))$ as the fertility rate at age x for cohort born in τ, we have that

$$CTFR(\tau) = R \int_{\mathcal{R}^+} h(x; \mu(\tau), \sigma(\tau), \gamma(\tau))dx = R. \tag{9}$$

Given that $h(\cdot)$ is a probability density function, its integral over R^+ is inherently equal to 1, regardless of the specific values assigned to $\mu(\tau), \sigma(\tau), \gamma(\tau)$. However, when calculating the PTFR for the calendar year t, its value is derived from a 'mixture' of cohort age-specific fertility rates. Specifically, each period-age-specific fertility rate at age x in calendar year t can be expressed as $h(x; \mu(t-x), \sigma(t-x), \gamma(t-x))$. Due to the variation in the parameters of $\mu(t-x), \sigma(t-x), \gamma(t-x)$ among cohorts, the integral of $h(\cdot)$ no longer yields 1:

$$PTFR(t) = R \int_{\mathcal{R}^+} h(x; \mu(t-x), \sigma(t-x), \gamma(t-x))dx \neq R. \tag{10}$$

Therefore, the PTFR(t) becomes distorted. However, we can use the fact that period-age-specific fertility rates can be written using Equation (7), and considering the adjusted rates defined in Equation (8) we get

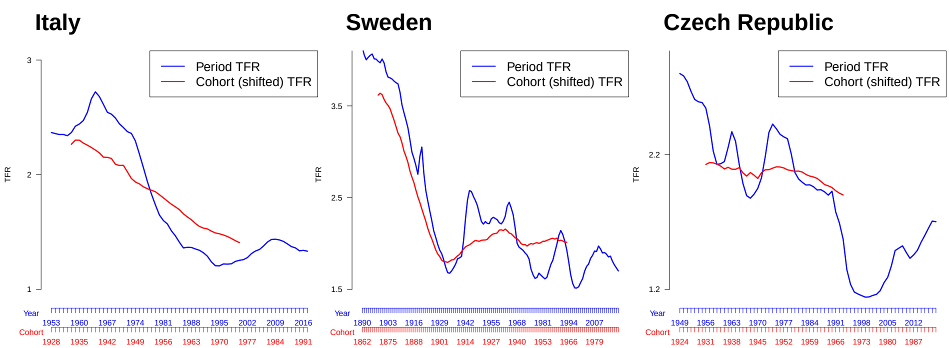
$$\widetilde{PTFR}(t) = R \int_{\mathcal{R}^+} \tilde{f}_x^t = R \int_{\mathcal{R}^+} h(x|\mu^{t_0}, \sigma^{t_0}, \gamma^{t_0})\lambda^{t_0} dx = R. \tag{11}$$

This means that we can adjust the age-specific fertility rate f_x^t getting the value that would have been observed if the timing of the fertility schedule had remained stable using Formula (8), and the sum of adjusted age-specific fertility rates provides the adjusted PTFR. This adjustment is possible with the estimates of parameters μ, σ, and γ for every cohort since t_0 up to t, which is a major limitation because the data spanning from $t_0 - 15$ up to $t + 50$ are required. However, this result can be used to show how cohort changes in tempo parameters – not only mean but also variance and skewness – impact period quantum. In the following Section, we use this result to assess the effect of changing the timing of cohort fertility schedules on both period fertility schedules and PTFR.

3. Data

We examined fertility patterns in three distinct countries – Italy, Sweden, and the Czech Republic – using data sourced from the Human Fertility Database, which provides cohort and period fertility rates for ages 12 to 55. For Italy, period data covers the years 1954 to 2019 (the latest available at the time of data retrieval), while cohort data pertains to women born between 1939 and 1970. Sweden boasts extensive data series, with period information dating back to 1891 and the first available cohort being individuals born in 1876. The particularly long time series of Swedish cohort data is crucial for effectively implementing the method. This extensive data coverage, comprising 94 cohorts, enables us to gather the necessary information on mean, scale, and shape to adjust the total fertility rate for a specific calendar year. Specifically, to perform this adjustment, we require the fertility schedules for the 44 cohorts that are fertile within that year. Conversely, for the Czech Republic, we have PTFR data from 1950, and the earliest available cohort is from 1935. Figure 1 illustrates the PTFR and CTFR trends in these countries.

Figure 1: Time trend of PTFR and CTFR (shifted by the mean age at birth) in Italy, Sweden, and Czech Republic



Source: Human Fertility Database, data downloaded on July 9, 2011.

These nations were selected because they exhibit different cases where the cohort trend is not mirrored in the period trend. This observation makes it particularly intriguing to discern which characteristic(s) of the cohort distribution influences the observed period trend. In Italy, the CTFR exhibits a consistent decrease, yet this trend does not align with the PTFR, which displays an increase around the 1960s (baby boom), followed by a rapid decline and a slight recovery around 2010. In Sweden, the PTFR trend fluctuates, a pattern not mirrored in the CTFR trend, which exhibits considerably less variation. The period and cohort trends coincide only in the initial 30 years of the 20th century, showcasing a rapid decline in both. In the Czech Republic, the CTFR demonstrates an almost stable trend between 1955 and 1990, in contrast to the PTFR, which is relatively unstable during the same period. Even more interesting is the time pattern of the tempo coefficients of the skew-normal Model (3) that have been estimated for available cohorts via nonlinear least squares. These patterns are reported in Figures A-1, A-2, and A-3 (in Appendix A). In Italy, there is a rising mean age at childbirth, accompanied by an increasing variance and decreasing skewness. In Sweden, the mean age at childbearing decreases, in contrast to Italy, while variance and skewness undergo substantial changes, especially between 1900 and 1920 cohorts. Notably, the skewness parameter drops very quickly and then rises back to the previous value in a very short time. This is likely an effect of fertility postponement during World War I, which, from a cohort perspective, affected the shape of fertility schedule. Finally, for the Czech Republic, both mean age and skewness do not change dramatically, while variance first rises and then drops. Other data could have been used here, but these three countries provide good examples of possible tempo effects that could lead to quantum distortion. It would be interesting to consider other countries like the United States or United Kingdom, where it has been noted the emergence of an additional hump (see, for instance, Chandola, Coleman, and Hiorns 1999; Mazzuco and Scarpa 2015). A focus on the United States would be also interesting to compare our results with those by Schoen (2022) and Schoen and Hargens (2023). However, including the United States or a country with a similar fertility schedule in the analysis is a bit problematic: The additional mode or hump we note in these countries cannot be fitted well neither by the skew-normal distribution nor by the Peristera–Kostaki one. Chandola, Coleman, and Hiorns (1999) suggest a mixture of Hadwiger distributions, whereas Mazzuco and Scarpa (2015) suggest an extended version of skew-normal distributions, flexible generalized skew normal (FGSN), to deal with this data, but employing these solutions would lead us to several issues:

1. Both the FGSN and mixture of Hadwiger distributions would be over-parameterized in most of the cases (where there is no hump or additional mode).
2. When an additional mode/hump appears, we have at least one additional parameter. In the case of FGSN, we have a second shape parameter that affects the first one. So the interpretation of parameters becomes less straightforward.

The trends described above can be used as a base to simulate some scenarios where the quantum parameter remains fixed and the tempo ones vary.

4. Simulation studies

To investigate the transient behavior of period fertility when cohort parameters change over time, we conducted a simulation study aimed at understanding how alterations in mean, variance, and skewness observed at the cohort level affect PTFR bias. Unlike an analytical, moments-based approach (e.g., Keilman 2006; Ryder 1956), which accurately predicts equilibrium values of period fertility, simulation is essential here to capture the effects of shifting cohort parameters. To construct realistic scenarios, we simulated the distribution of cohort-specific fertility rates using the estimates for Italy, Sweden, and the Czech Republic, as described earlier. This yielded three sets of scenarios, one for each country.

In the first set of scenarios labeled Italy, we simulate 150 cohorts to explore the impact of changes in mean, variance, and skewness on PTFR bias. The initial 50 cohorts maintain fixed parameter values as specified in Table 1.

Table 1: Definition of scenarios

Italy				
Scenarios	**R (fixed)**	**μ**	**σ**	**γ**
1	2.025	26.80 – 30.97	5.1 (fixed)	0.60 (fixed)
2	2.025	26.80 (fixed)	5.1 – 6.1	0.60 (fixed)
3	2.025	26.80 (fixed)	5.1 (fixed)	0.60 – –0.22
4	2.025	26.80 – 30.97	5.1 – 6.1	0.60 (fixed)
5	2.025	26.80 – 30.97	5.1 (fixed)	0.60 – –0.22
6	2.025	26.80 – 30.97	5.1 – 6.1	0.60 – –0.22
Sweden				
Scenarios	**R (fixed)**	**μ**	**σ**	**γ**
1	1.858	29.67 (fixed)	7.40 (fixed)	0.818 – –0.0005
2	1.858	29.67 (fixed)	7.40 – 5.88	0.818 – –0.0005
3	1.858	30.05 – 29.31	7.40 – 5.88	0.818 – –0.0005
Czech Republic				
Scenarios	**R (fixed)**	**μ**	**σ**	**γ**
1	2.022	25.34 (fixed)	5.041 – 4.083	0.911 (fixed)
2	2.022	25.34 (fixed)	5.041 – 4.083	0.911 – 0.793
3	2.022	25.34 – 24.23	5.041 – 4.083	0.911 – 0.793

The equilibrium is then perturbed by linearly changing the value of one or more parameters over the next 50 cohorts. This means that the transition from the initial value to the final value is achieved by dividing the range equally over the chosen transition period, which in this case is 50 years. The final 50 cohorts are simulated with the new, constant parameter values (see again Table 1). Specifically, we first vary each parameter individually, then examine the combined effects of simultaneously changing the mean and variance as well as the mean and skewness, and finally, investigate how alterations in all three components collectively influence PTFR bias. This gradual transition from 'old' to 'new' values across 50 cohorts, achieved through a linear change, allows us to effectively demonstrate the equilibrium before and after the transition.

In the second set of scenarios (Sweden) we simulate a rapid drop in skewness while keeping the other parameters fixed. We then explore the effects of combining this skewness drop with a decrease in variance, and finally, we simulate the simultaneous changes in mean, variance, and skewness. In this set, the transition occurs over just 5 cohorts. To illustrate the equilibrium before and after this rapid change, we add 50 cohorts before and after the transition, with parameters fixed at their initial and final values, as specified in the Table 1. Consequently, the total number of cohorts simulated in this scenario amounts to 105.

Finally, in the Czech Republic set of scenarios, we focus on understanding PTFR bias when only the variance component changes, and we evaluate the additional effects when a slower decline in mean and symmetry is introduced. In this scenario, a rapid decrease in variance over 5 cohorts is combined with a slower decline in mean and skewness (the range of the two last parameters is very small, while the variance one is wider). As in the previous scenario, 50 cohorts are added before and after the transition, resulting in a total of 105 cohorts simulated: the first 50 with fixed initial values and the last 50 with fixed final values.

All parameters used are detailed in Table 1. For each set of parameters, we calculate the PTFR for each year. Results are shown in Figures 2, 3, and 4.

Figure 2: PTFR in six simulated scenarios, Italy

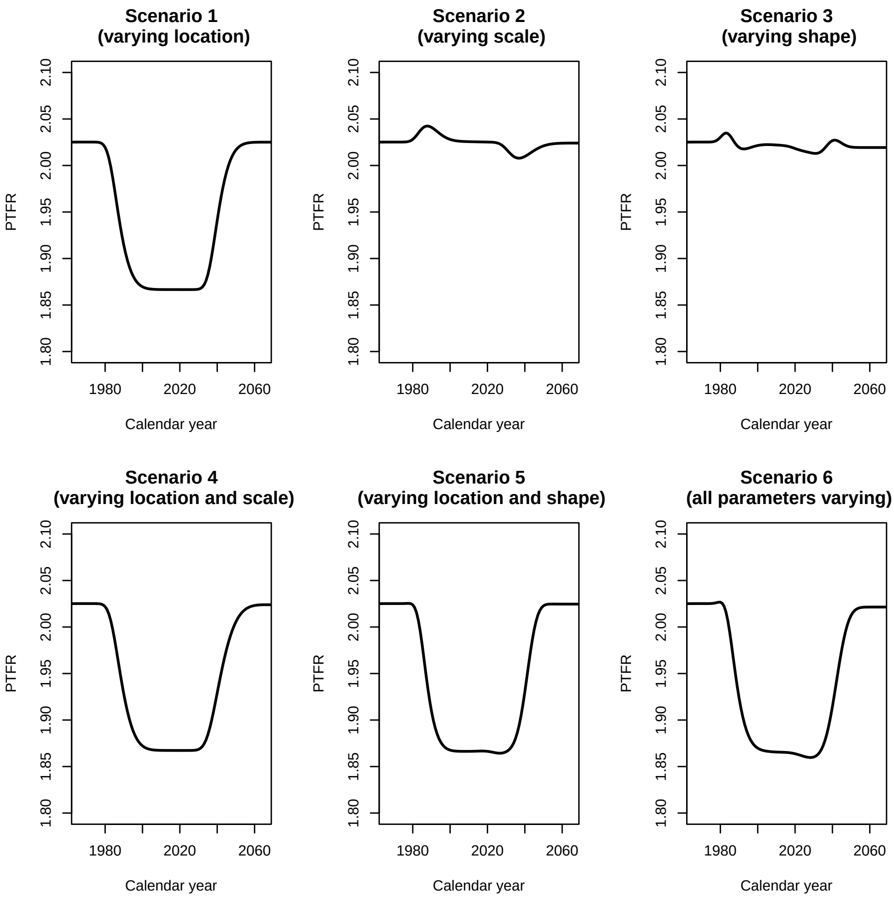


Figure 3: PTFR in three simulated scenarios, Sweden

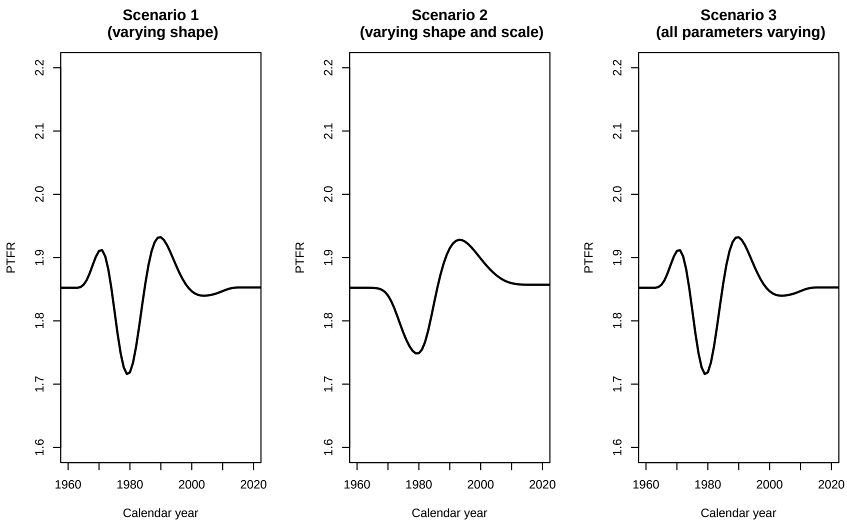
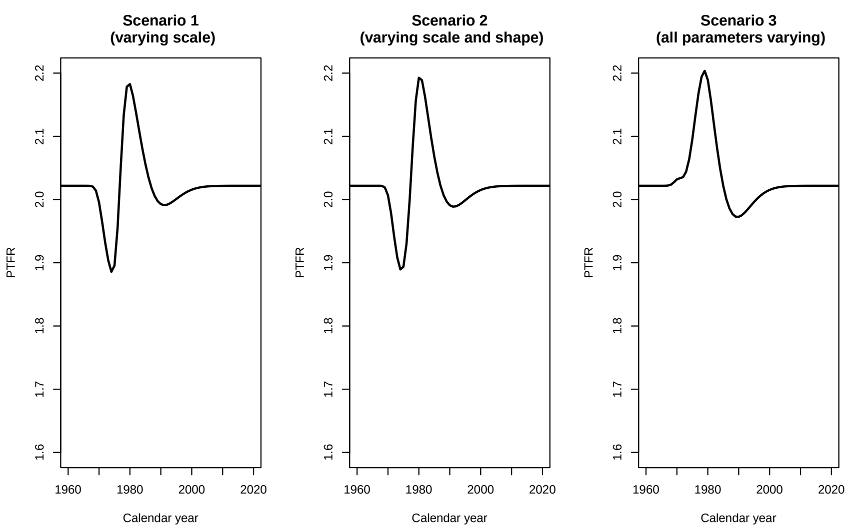


Figure 4: PTFR in three simulated scenarios, Czech Republic



If we look at the Italy scenarios, we find that the location shift determines a distortion in PTFR, that drops from 2.02 to 1.87 and then get back at its original value (2.03). More interestingly, in Scenarios 2 and 3 (where only shape and scale parameters change) the modifications to PTFR are negligible. This is an important result, as it apparently indicates that tempo distortion is mainly brought about by fertility postponement, while shape and scale changes of births age distribution can be disregarded, confirming what is already shown by Kohler and Philipov (2001) and Yi and Land (2001). However, if we turn to the Sweden sets of scenarios, we find, in contrast, that shape and scale parameters change (Scenarios 1 and 2) heavily impact PTFR, which fluctuates between 1.62 and 2.00 before getting back to its original value (1.86). The drop in mean age at child bearing (Scenario 3) actually moderates these fluctuations. Similarly, in the third set of scenarios, Czech Republic, where the CTFR is set at 2.02, a rapid scale variation (Scenario 1) makes the PTFR range from 1.89 to 2.18. If a change in shape parameter is also included (Scenario 2), the result is very similar, while, once again, the change in location moderates these fluctuations. So it turns out that a rapid change in the shape and scale of fertility schedules also determines a relevant tempo effect. If such changes are less rapid, the tempo effect is milder. Interestingly, it might also happen that the location parameter variation mitigates the tempo effect brought about by scale and shape.

We ran further simulations to see how fast the scale and shape variation should be to determine a significant tempo distortion in the PTFR. Setting the value of the CTFR constant at 2.1 and keeping the other coefficients fixed, each parameter varies individually considering different time ranges. The intensity of the variation is given by the coefficient $\beta_{(\cdot)}$, which indicates a linear annual increase/decrease of the parameter under consideration. For example, when $\beta_{(\cdot)} = 0.692$, it means that the value of the parameter increases by 0.692 per cohort. In our simulations, we calculate the mean age at childbirth (parameter μ) shift from age 26 to 35 with an increasing pace: We started setting a value of $\beta_\mu = 0.06$, meaning that it took 150 cohorts for μ to get from 26 to 35, then gradually increased β_μ up to a value of 1 (only 10 cohorts to get from starting to final value). Similarly, for the standard deviation, we increase σ from 4 to 6, progressively reducing the transition time: from 150 cohorts involved down to 10 ($0.013 \leq \beta_\sigma \leq 0.222$). As for the shape parameter, γ, we decrease it from 0.9 (right skewed) to –0.5 (left skewed), still with the same declining transition time ($-0.156 \leq \beta_\gamma \leq -0.009$).

In Figures A-6, A-7, and A-8 (in Appendix A), we show the maximum of the ratio between the CTFR (always fixed at 2.1) and PTFR in relation to the values of $\beta_{(\cdot)}$.

Increasing the position parameter (μ) always results in an underestimation of the PTFR, and its bias is larger the more the speed of the transition increases. Particularly when the changes are small and constant, the alteration affects all years of the simulated interval, but it is smaller (instead of 2.1, 1.98 is observed). If the change involves only few cohorts but is rather rapid, the calendar years affected by the error are fewer, but the distortion is larger. Increasing the standard deviation (σ) from 4 years to 6 years produces first an overestimate of the PTFR (maximum 2.3) and then an underestimate (minimum 1.97). In this case, even if the distortion is smaller (but the range of variation is also smaller), the speed of change still plays a central role: Very quick transitions produce larger distortions, although they involve fewer years. The change in symmetry due to the shape parameter (γ) also results in an error in the PTFR. When the transition occurs very quickly (10 cohorts involved), the PTFR is underestimated and overestimated by a maximum of 0.1 children per woman in a time span of about 30 years. As the transition period increases, the bias becomes less and less influential, even involving more calendar years.

For all three parameters, the relationship between the speed of change, given by the parameter $\beta_{(\cdot)}$, and the maximum distortion, which is given by the maximum of the ratio between CTFR and PTFR, was studied. In each case, the link between the two components can be well approximated by a linear regression model, of which the coefficient of determination results in 0.995 for μ, 0.971 for σ and 0.982 for γ.

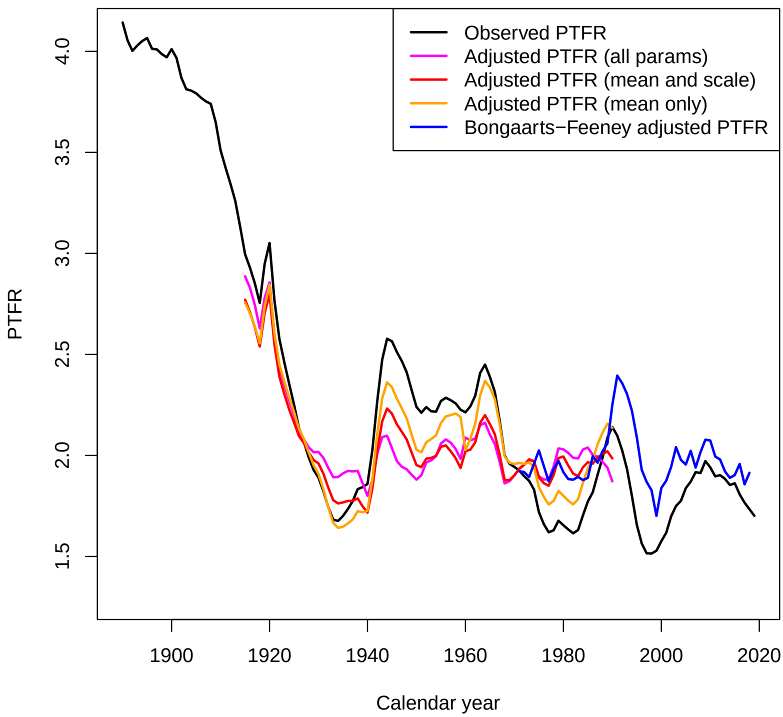
Do these results contradict the works by Kohler and Philipov (2001) and Yi and Land (2001), showing that scale and shape changes in fertility schedules only mildly affect PTFRs? Not necessarily, because in both these works period shape and scale shifts are considered, while here the focus is on cohort shape and scale changes, and the relationship between them is not straightforward. In order to show this, we fit the skew-normal model with the period age-specific fertility rates obtained from the simulations of the Sweden set and estimate the related parameters μ, σ, and γ. In this way we can show how the cohort timing parameters are related with period ones. In Figure A-4, we can see the effect of rapidly decreasing cohort skewness on the PTFR and the period parameters μ, σ, and γ. It emerges that the period skewness is only weakly decreasing but also mean and variance vary. In Figure A-5, instead, the decline of cohort variance is matched with a similar drop in period variance, but also in this case, the period mean is affected. Although variations in the scale and shape of cohort fertility schedules can significantly distort the PTFR, this bias appears to be almost entirely reflected in the shift in the mean of the period fertility rates distribution.

These results demonstrate a strong connection between the proposed method and the relationships made explicit by Keilman (2006), originally derived from Ryder (1956). The author has shown that the PTFR and the moments of the period fertility schedule can be expressed, via Taylor expansion, as a function of cohort schedule moments. In the Appendix B, we provide calculations that illustrate the link between the moments-based method and some of the results obtained through simulation. As the adjustment discussed in Section 2, the moments-based method can accurately determine the PTFR value for the new equilibrium but cannot predict PTFR values during the transition phase. Our simulations reveal that overshoots and undershoots occur during the transition, and these effects are more pronounced with steeper changes.

5. Application on real data

The method described in Section 2 is now applied to real-world data, shedding light on the practical implications of the impact of the three cohort components (mean, variance, and skewness) on the PTFR distortion. We chose data from Sweden, for which we have complete cohort fertility schedules from the generation born in 1876 to 1970. A long enough time series of cohort data is necessary to implement the method, as we need to know the mean, scale, and shape of fertility schedules for 44 cohorts to adjust the TFR of one period. Note that we can choose whether to adjust only for the mean or also for shape and scale parameters (see Figure 5).

Figure 5: Sweden observed PTFR and adjusted values



Source: Observed PTFR comes from Human Fertility Database.

The length of these series is smaller than the one drown for the observed PTFR. Implementing the proposed adjustment method necessitates cohort information, but for older cohorts, fertility rates at younger ages are unknown, and the more recent cohorts have not yet reached exhaustion. In these cases, it is therefore not possible to estimate the model parameters. As a result, the 'all params,' 'mean and scale,' and 'mean only' series are shorter and range from 1915 to 1990. The adjustment for the first year requires cohort data from 1870 to 1902, while adjusting the 1990 PTFR we need information on women born from 1936 to 1968. Before and after these two calendar years, not all the necessary data are available.

These adjustments are also compared with the Bongaarts–Feeney adjustment, which requires knowledge of birth order, available from 1971 until 2018. First, we should keep in mind that the two methodologies diverge at their starting points: Bongaarts and Feeney adopt a period perspective, whereas our approach is rooted in a cohort perspective. Second, the primary objective for both authors is distinct: Bongaarts and Feeney aim to attain a PTFR adjustment through the utilization of birth-order-specific fertility rates. In contrast, our analysis centers around comprehending how changes observed at the cohort level manifest in the distortion of the average number of children per woman in calendar year t. From our standpoint, the adjustment resulting from the model serves as a necessary consequence for our analytical objectives. However, considering that the BF adjustment is the widely adopted method for PTFR correction, and our proposed method ultimately also serves as an adjustment of the PTFR, we find it pertinent to compare the results derived from these two methodologies. Figure 5 shows that actually adjusting for shape and scale matters: The adjusted value of PTFRs, taking into account the variation of all parameters, is different from those based on only mean variation. Such a difference is particularly pronounced between 1940 and 1950, consistently with a sharp variation of shape and scale parameters for cohorts born between 1910 and 1920. More interestingly, it can be seen that the BF-adjusted PTFR is closer to the values we get when we take into account the changes of all tempo parameters (i.e., mean, shape, and scale changes) rather than to only partially (i.e., considering only mean shifts) adjusted PTFR. This is further evidence that, from a period perspective, taking into account only location shifts is an effective strategy.

6. Discussion and conclusions

Since the work in which Bongaarts and Feeney (1998) propose a simple method to adjust PTFRs from tempo distortions, there have been several contributions exploring the potentialities and drawbacks of the BF method. One suggested pitfall is that this adjustment takes into account only the temporal shift of fertility schedules, disregarding variations in terms of scale and shape (see van Imhoff 2001). However, Kohler and Philipov (2001) and Yi and Land (2001) show that the BF method is generally robust to scale and shape changes. All these works, however, take a period perspective, while Goldstein and Cassidy (2014) state that there are several advantages to considering a cohort approach, one of these is that it is more consistent with a life-course approach. We therefore model tempo fluctuations from a cohort perspective and find out some interesting results. First, as demonstrated by Keilman (2006) and Ryder (1956), we have shown that variations in the scale and shape of cohort fertility schedules can significantly impact PTFR estimates. However, in addition to the moments-based method, our methodology highlights the fluctuations experienced by the PTFR as it approaches its equilibrium point. The Sweden and Czech Republic sets of simulations show that by changing shape and scale parameters, and keeping the mean and the CTFR fixed, we can obtain relevant fluctuations of the PTFR (see Figures 3 and 4). Second, the impact of cohort changes on the PTFR depends on how fast they are: If shape and scale vary slowly (see the Italy sets of simulations), there is only a negligible impact on PTFR distortion. But if these changes are taking place rapidly, as they did, for instance, in Sweden for cohorts born between 1900 and 1920, and in the Czech Republic for cohorts born between 1945 and 1950, then the tempo distortion is much higher. Third, the shape and scale fluctuations of cohort fertility schedules do not bring about the same or even a similar shape and scale fluctuations of period fertility rates. In particular, we have seen that a fast and large drop in cohort fertility skewness leads to an only mild change of period fertility skewness. On the other hand, these variations have an effect on the mean of period schedules. In theory, an observed postponement or anticipation of the mean age at childbearing of period fertility can be the result of the variation only of shape and scale at the cohort level.

These results explain why it has been found that the BF method is robust to shape and scale fluctuations, although it takes into account only temporal shifts: Scale and shape matter in terms of tempo distortion, but their cumulated effect on period fertility schedules is better caught by the mean rather than variance and skewness. Figure 5 actually shows that the BF method is close to the adjustment we can make with the cohort model considering not only the temporal (i.e., mean) shifts but also the scale and shape variations.

Our findings unequivocally demonstrate the intricate nature of fertility, shaped by a confluence of period and cohort factors, as elucidated by Schoen (2022). Analyzing the PTFR reveals conspicuous fluctuations, signifying that events experienced by women during their fertile years significantly influence their reproductive decisions, either hastening or delaying the choice to have children. These decisions manifest in the distribution of fertility rates across cohorts. Conversely, when examining the CTFR, we generally observe smoother trends, indicating a strong cohort effect on fertility intentions (desired number of children). This suggests that, in the long run, neighboring cohorts exhibit similar fertility behaviors, influencing the corresponding period trend. The proposed method offers the advantage of unraveling these intricate relationships, shedding light on the specific cohort components driving observed trends in each period.

Furthermore, incorporating cohort-based fertility measures allows for a more comprehensive understanding of the sustained impacts of population policies. Fertility measures based solely on period data may fall short of capturing the enduring effects of policies, especially during periods of rapid social, economic, and demographic change. Evaluating fertility interventions necessitates distinguishing between anticipatory trends and genuine increases. The proposed adjustment method accommodates these complexities. For instance, the recent decline observed in Sweden may be attributed to alterations in the shape and scale of cohort data rather than an actual reduction in the average number of children per woman. The methodology's sole drawback is the requirement for multiple completed cohorts, corresponding to each age within the fertile window, to provide adjustment for the total fertility rate by period. Nonetheless, it merits recognition for discerning the impact of each individual component (e.g., position, scale, and shape) of time distortions without relying on assumptions to calculate the adjustment.

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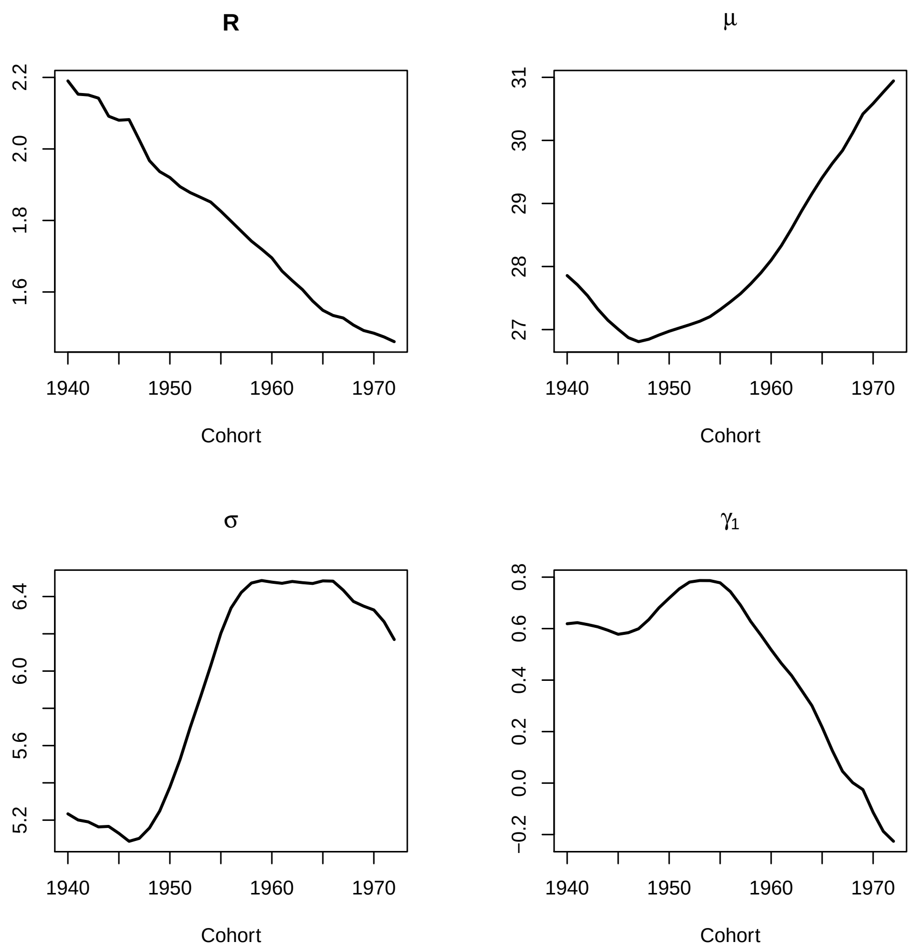
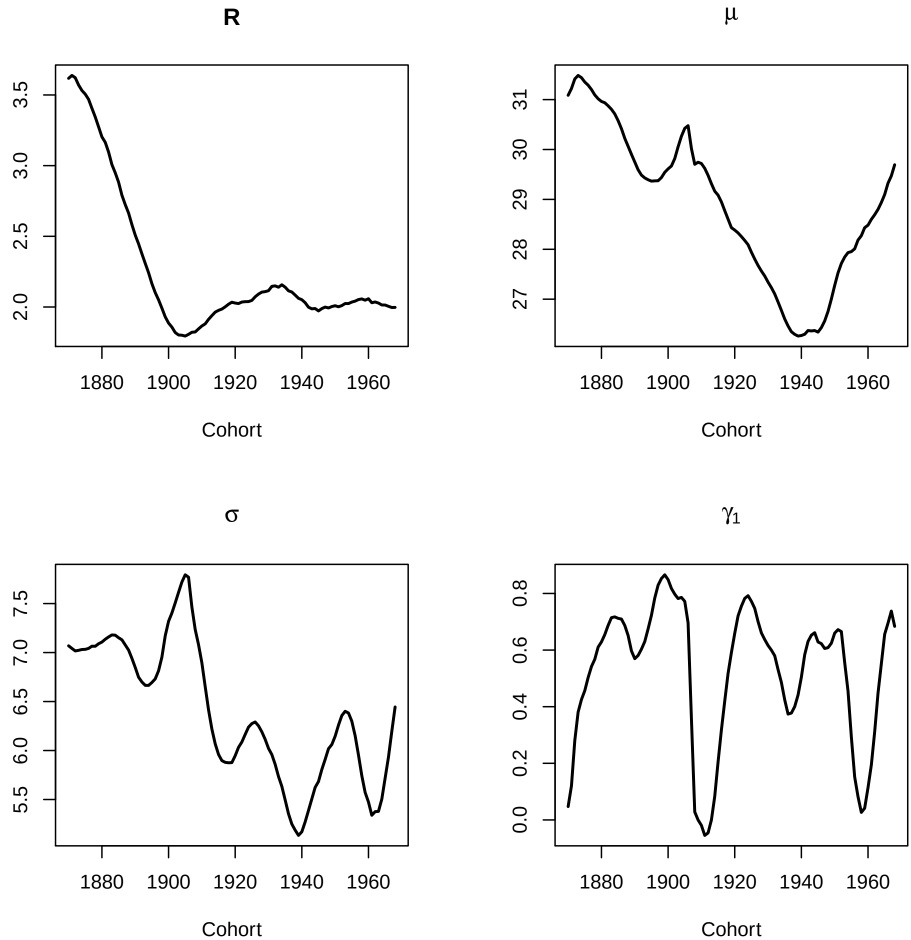
Appendix A

Figure A-1: Estimate of skew-normal model parameters for Italian cohorts 1940–1972



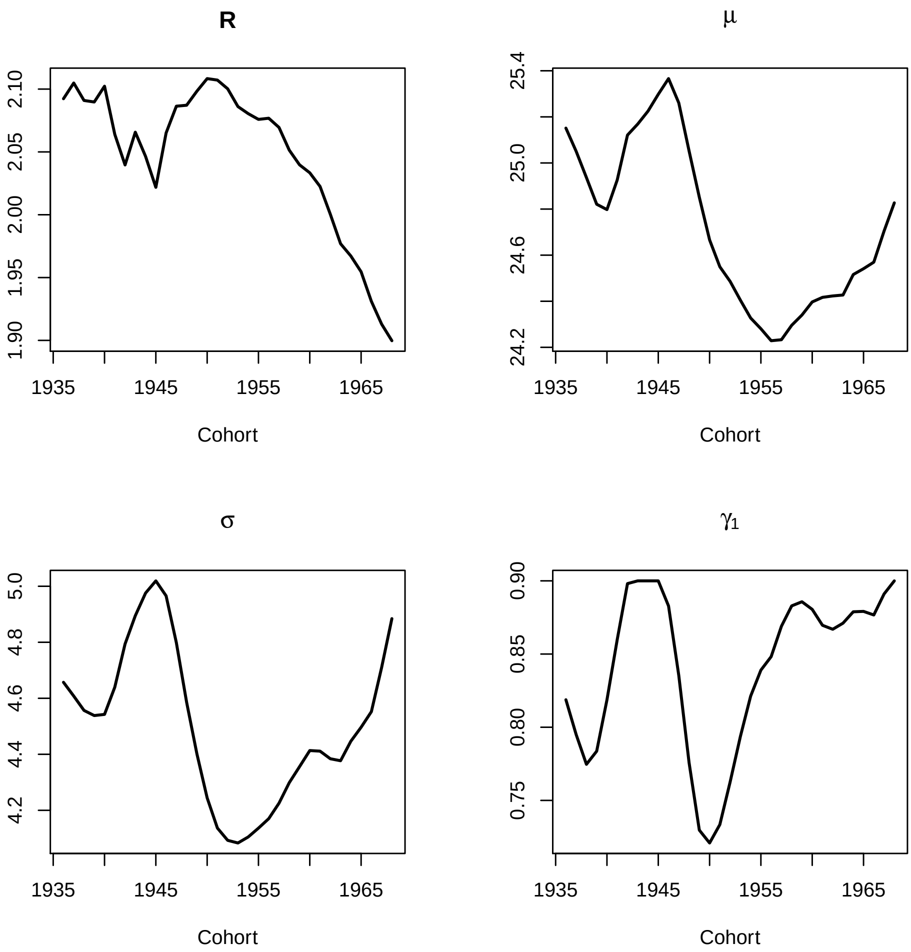
Source: Human Fertility Database (2020).

Figure A-2: Estimate of skew-normal model parameters for Sweden cohorts 1876–1972



Source: Human Fertility Database.

Figure A-3: Estimate of skew-normal model parameters for Czech Republic cohorts 1935–1972



Source: Human Fertility Database.

Figure A-4: Estimate of skew-normal model parameters for a simulated scenario

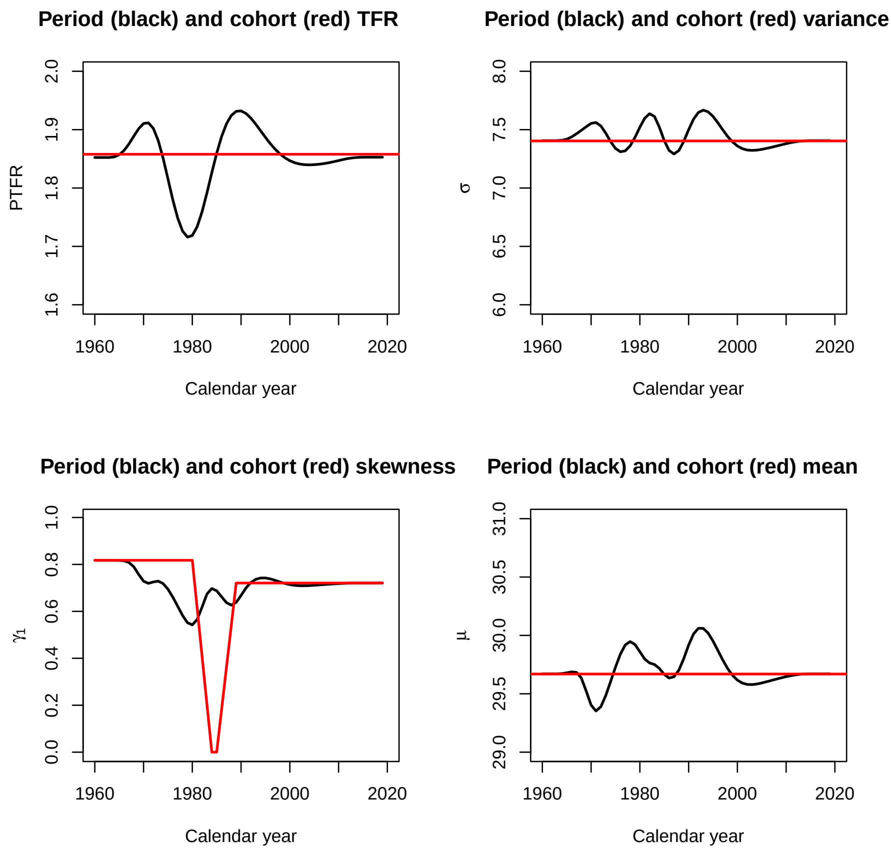


Figure A-5: Estimate of skew-normal model parameters for a simulated scenario

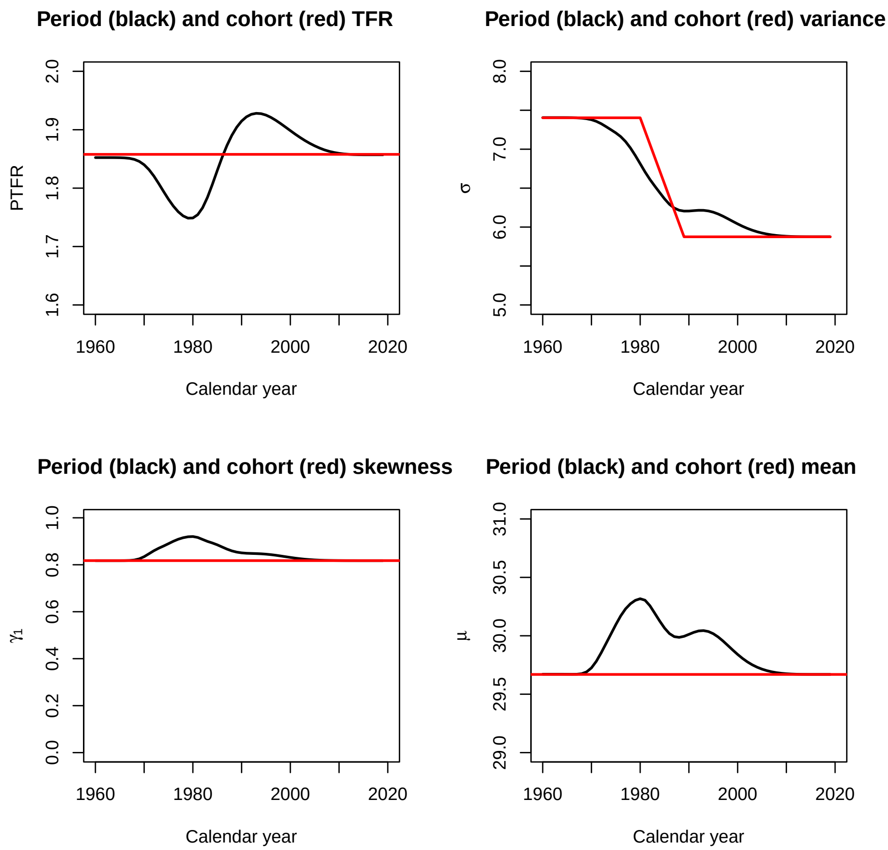


Figure A-6: Tempo distortion of PTFR in function of rapidity of location parameter shift

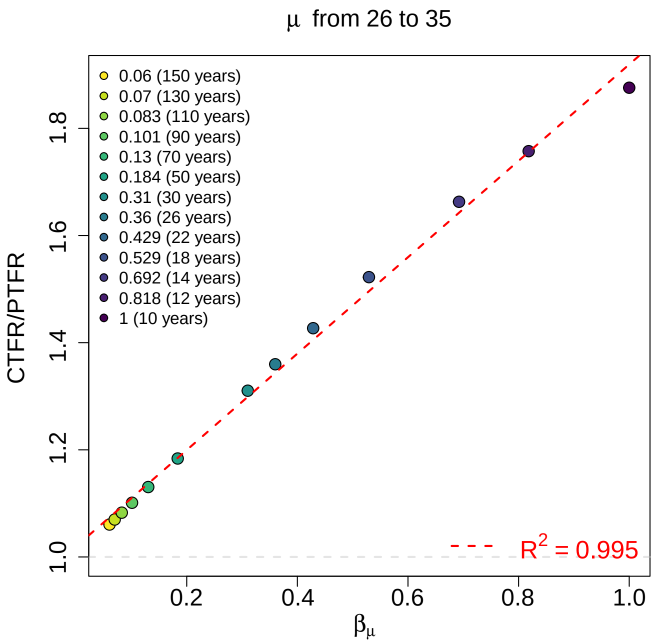


Figure A-7: Tempo distortion of PTFR in function of rapidity of scale parameter shift

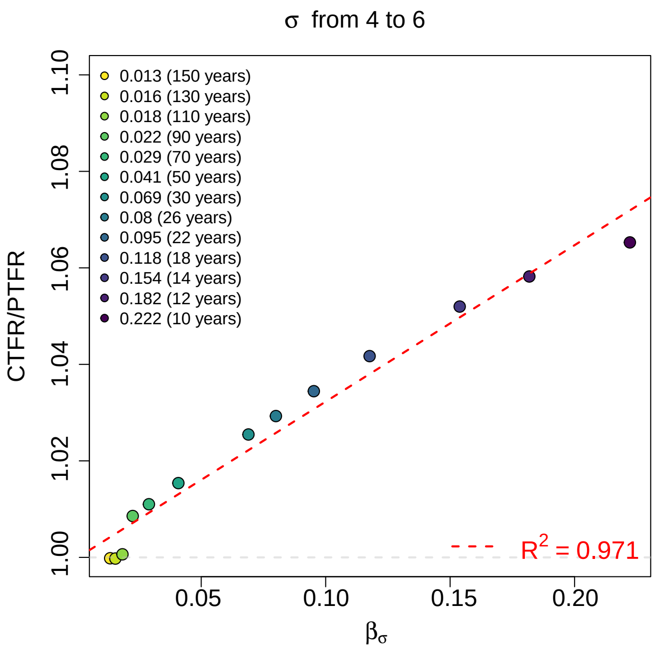
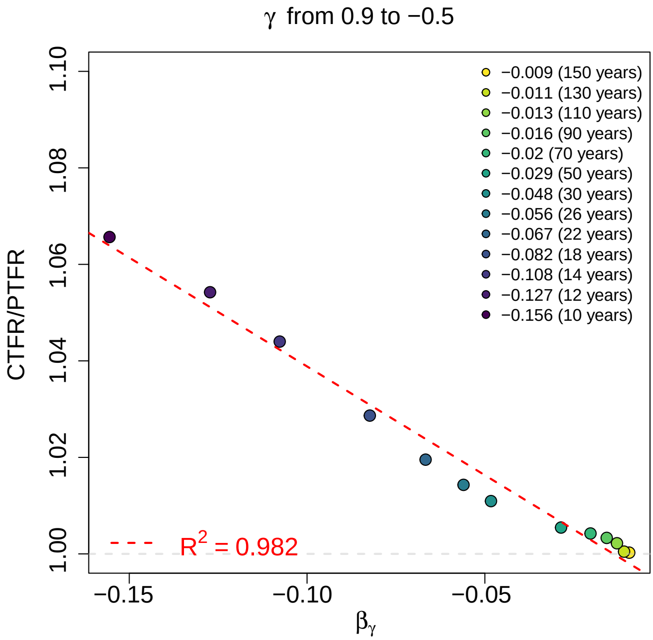


Figure A-8: Tempo distortion of PTFR in function of rapidity of shape parameter shift



Appendix B

Let $V_k(t)$ and $W_k(t)$ denote the moments of order k for the schedule of age-specific fertility rates for calendar year t and for women born in year t, respectively. Moments of order zero represent the period TFR and the cohort TFR: $V_0(t) = PTFR(t)$ and $W_0(t) = CTFR(t)$, respectively. A Taylor series approximation of the period TFR, as shown by Keilman (2006), gives

$$V_0(t) = W_0(t) - W_1^{'}(t) + \frac{1}{2}W_2^{''}(t) - \frac{1}{6}W_3^{'''}(t) + \cdots \,, \tag{12}$$

where the prime denotes derivation with respect to time. It is important to note that the moments are non normalized and non central.

Throughout the paper, we assume the CTFR constant, while the mean age of the cohort fertility schedule $\mu(t)$, the cohort-standard deviation $\sigma(t)$, and the cohort-skewness $\lambda(t)$ are either constant or a linear function of time. Therefore,

$$\begin{aligned}
W_1^{'}(t) &= W_0\mu^{'}; \\
W_2^{''}(t) &= \left\{W_0\left[\sigma^2(t) + \mu^2(t)\right]\right\}^{''} = 2W_0\left[(\sigma^{'})^2 + (\mu^{'})^2\right]; \\
W_3^{'''}(t) &= \left\{W_0\left[\gamma(t)\sigma^3(t) + 3\sigma^2(t)\mu(t) + \mu^3(t)\right]\right\}^{'''} = \\
&= 6W_0\left[3\gamma^{'}\sigma(\sigma^{'})^2 + \gamma^{'}(\sigma^{'})^3 + 3\mu^{'}(\sigma^{'})^2 + (\mu^{'})^3\right].
\end{aligned} \tag{13}$$

When $\mu(t)$ is a linear function of the time, while the CTFR is constant and both σ e γ are fixed, we obtain

$$PTFR(t) = PTFR = \frac{CTFR}{1+\beta_\mu}, \tag{14}$$

where $\beta_\mu < 1$ is the slope of the mean age.[4] For example, in Italy, Scenario 1, where only μ varies, using CTFR = 2.025 and β_μ = (30.97-26.80)/50 = 0.0834, Equation (14) indicates PTFR = 1.87 children per woman on average, consistent with the new equilibrium value shown in Figure 2, Scenario 1. Additionally, the moments-based method predicts that the distortion CTFR/PTFR equals $1 + \beta_\mu$, a result confirmed by Figure A-6.

[4] For $\beta_\mu > 1$, the Taylor series does not converge.

In Italy, Scenario 2, where only σ is a linear function of time, Equation (13) gives

$$\begin{aligned} W_1^{'}(t) &= 0; \\ W_2^{''}(t) &= 2W_0\beta_\sigma^2; \\ W_3^{'''}(t) &= 6W_0\gamma\beta_\sigma^3; \\ PTFR &= CTFR\left[1+\beta_\sigma^2-\gamma\beta_\sigma^3\right]. \end{aligned} \tag{15}$$

With CTFR = 2.025, $\gamma = 0.6$, and $\beta_\sigma = (5.1 - 6.1)/50 = -0.02$, the moments-based method predicts PTFR = 2.026, the same as Figure 2, Scenario 2 (the small ripples in the graph are due by transient behavior during the transition from an old regime to a new one). Moreover, since, the distortion CTFR/PTFR is the inverse of a third-degree polynomial in β_σ, for small values of γ and β_σ, the graph of this ratio approaches a straight line, as confirmed by Figure A-7.

In the Czech Republic, Scenario 1 follows the same assumptions as Italy, Scenario 2. Using CTFR = 2.022, $\gamma = 0.911$, and $\beta_\sigma = (4.083 - 5.041)/5 = -0.19$, the PTFR becomes 2.109. However, the new equilibrium calculated by moment-based method is difficult to observe in Figure 4, Scenario 1, due to the rapid transition of σ, which causes strong distortions over a short period. These first three examples show that the moment-based method aligns well with our simulations, provided that changes occur over longer intervals, although it cannot predict the values during the transition phase, which are particularly pronounced when change occurs rapidly.

Neglecting terms of order four or higher, when CTFR, μ, and σ are held constant and only γ varies linearly (as in Italy, Scenario 3), we have $W_1^{'}(t) = W_2^{''}(t) = W_3^{'''}(t) = 0$, resulting $PTFR = CTFR$. Figure 2, Scenario 3 shows a ratio close to 1 ($\beta_\gamma = (-0.22 - 0.6)/50 = -0.0164$). A similar ratio is also observed in Figure A-8, when the change occurs over a long interval (β_γ is small). In Figure A-8, we observed a gradual shift from the value 1 as the transition takes place more quickly. The discrepancy arises because the graph displays the maximum ratio of CTFR to PTFR: When γ changes rapidly (steep slopes), the transient behavior of PTFR causes overshooting and undershooting.

In conclusion, the moments-based approach, as proposed Keilman (2006), yields results comparable to our simulations when changes occur over extended periods. However, the simulations presented uniquely capture transient behavior, revealing that overshoot and undershoot effects become more pronounced with steeper regime changes.