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Research Material

Modifying model life tables to derive mortality curves for countries with excess mortality

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Contents

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Modifying model life tables to derive mortality curves for countries with excess mortality

Lina Maria Sanchez-Cespedes[1]

Abstract

BACKGROUND
Model life tables are valuable tools for filling gaps in mortality data when estimates are only available for specific age groups, and have been used in many countries. However, their relevance has declined, as they fail to account for cause-specific mortality, leading to biased results in populations with significant differences from the reference data, such as countries with high levels of violence or road accidents.

OBJECTIVE
This study examines whether traditional model life tables can still be used to estimate mortality curves and their backward projection in countries with excess mortality due to external causes.

METHODS
We propose a simple method to adjust Coale–Demeny or UN model life tables in order to estimate mortality curves for countries with excess mortality. The method identifies the most appropriate model life table that reflects the mortality pattern without excess deaths and adjusts it to the actual one by considering external-cause mortality rates. This allows for estimating life expectancy differences with and without excess mortality.

RESULTS
We exemplify the method with the case of Colombia. The results show a difference in life expectancy at birth between no excess mortality and excess mortality, $\Delta e(0)_{\text{no-excess and excess}}$, of about –5 years in the 1990s, reaching –5.47 in 2000 and –2.39 in 2017.

CONCLUSION
This variation reflects key historical moments related to drug trafficking, the armed conflict, and shifts in government policy.

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CONTRIBUTION

This method estimates mortality curves that more accurately reflect the realities of countries with high external-cause mortality, providing a better understanding of its impact on life expectancy, and improving backward population projections.

1. Introduction

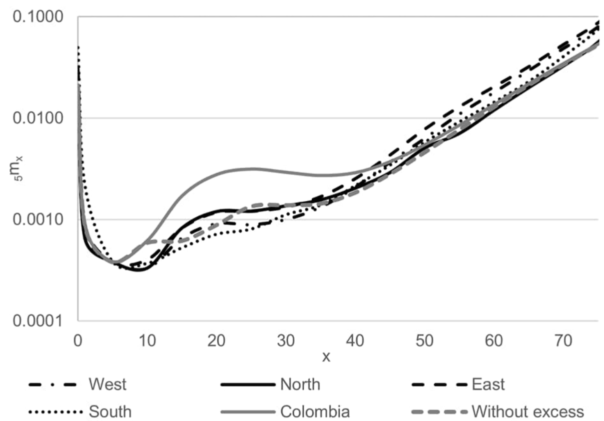
Although model life tables are statistical tools used to fill gaps in life table data when mortality estimates are only available for certain age groups, their use presents several challenges. These tables are frequently derived from data specific to certain regions or countries, making them less applicable to other populations or areas. For example, Coale–Demeny life tables are based on trends that researchers notice in life tables that use reliable data; thus, South model tables are based on life tables from Spain, Portugal, and Southern Italy (Coale and Demeny 1966). Consequently, model life tables typically do not account for cause-specific mortality rates, leading to biased results if significant differences exist between the studied population and the reference data (Buettner 2002; Moultrie et al. 2013). This is the case for countries with high levels of violence or road transport accidents (RTAs). Considering this gap, this study contributes to the literature by presenting a simple and intuitive method that adjusts Coale–Demeny or UN model life tables to estimate the mortality curves of countries with excess mortality due to external causes.

Excess mortality can generally be defined as all deaths that exceed what would be expected from a reference mortality pattern (Remund, Camarda, and Riffe 2018). Figure 1 illustrates the excess male mortality in Colombia for 2017. In this figure a notable adult mortality hump is observed between the ages of 15 and 40, standing out above the curves of the Coale–Demeny model life tables (the common point between the Colombian curve and Coale–Demeny's is the age range between 5 and 9 years, where there is no excess mortality). This hump is due to the excess death from homicides and RTAs, as Figure *2* shows. The male homicide rate in 2017 was 47.5 per 100.000 persons, and the death rate due to RTAs was 25.2.

For Colombia, 47.5 homicides per 100.000 men is a low rate compared to the historical record. From 1988 to 2004, due to the wars between and against drug cartels and armed conflict, male homicide rates consistently exceeded 100 per 100,000 inhabitants each year, while global rates remained around 10 per 100,000 (World Bank 2021a), (see Figure 3). Hence, Coale–Demeny model life tables were considered unreliable since they did not account for the impact of violence (DANE, DNP, CELADE, and CIID, 1989). Regarding RTAs, the death rate due to RTAs in Europe has been around

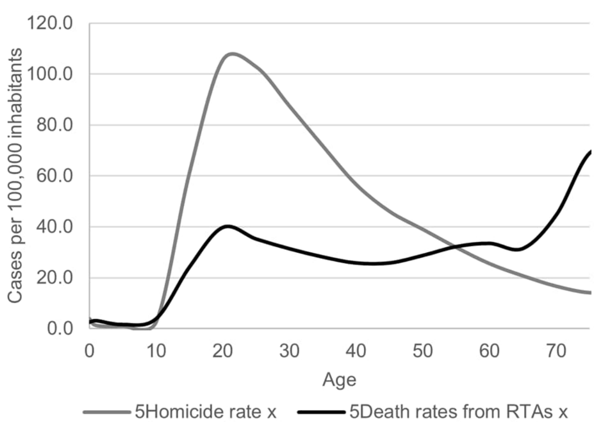
5 per 100,000 since 2013, while in Colombia it has been about 15 (World Bank 2021b). Nowadays, male homicide rates are twice the male death rates due to RTAs. However, in the 1990s it was between 4 and 7 times and in the 2000s between 3 and 5 times.

Figure 1: **Comparison of Coale–Demeny curves and Colombian mortality curve in 2017**



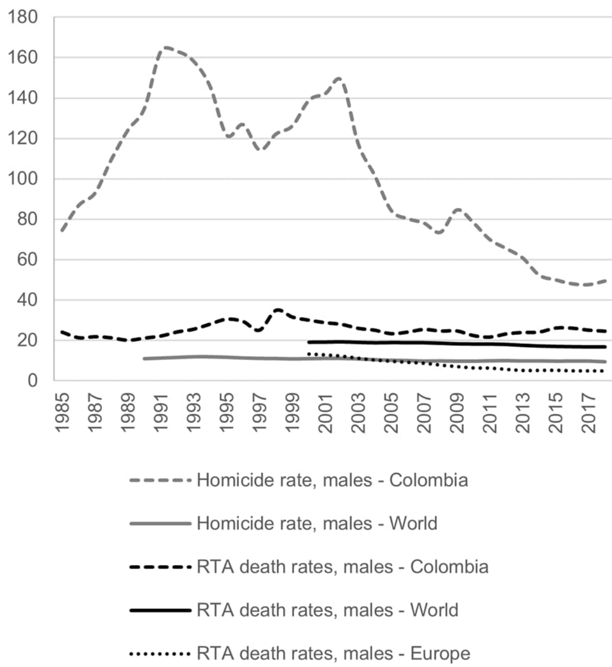
Source: Vital Statistics Public Data and population projections 2020, DANE's 2017 mortality estimates.

Figure 2: **Colombian $_5$Homicide rate$_x$ and $_5$Death rate from RTAs$_x$ versus age in 2017**



Source: Vital Statistics Public Data and population projections 2020, DANE's 2017 mortality estimates.

Figure 3: Historical male homicide and RTA death rates – Colombia and World



Source: Vital Statistics 1985–2019, Population projections DANE 2020, World Bank 2021.

Other authors have studied how specific causes of death contribute to the formation of the hump and the reduction in life expectancy, as well as how wars decrease life expectancy (Arriaga 1984; Remund, Camarda, and Riffe 2018; Lleras-Muney and Moreau 2022). Since the method proposed in this study identifies the model life tables (Coale–Demeny or the UN) that best approximate the mortality curve when there are no excess deaths, it allows us to estimate backward mortality projections and the difference in life expectancy at birth with and without excess mortality, $\Delta e(0)_{\text{no-excess and excess}}$.

We found that $\Delta e(0)_{\text{no-excess and excess}}$ was –2.39 years in 2017. The National Administrative Department of Statistics (DANE 2022a) calculated the Avoidable Years of Life Lost (AYLL) using Arriaga's method (1984) and data from 2017 to 2019. Homicides accounted for 1.67 AYLL, while road traffic accidents (RTAs) contributed 0.76, bringing the total to 2.43 AYLL. For the backward projection, we found that $\Delta e(0)_{\text{no-excess and excess}}$ reached values of about –5 years in the 1990s, –5.47 in 2000, and between –3.52 and –2.4 after that. The main reasons for this variation are explained in the discussion section.

2. Method

This section is divided into two parts. The first outlines the information required to implement the method, while the second provides a step-by-step explanation of its application, from the selection of the model life table to when the backward projection is obtained.

2.1 Required information

The information required to carry out the method is:

- ${}_5m_{x\ in\ Y,i}$ or ${}_1m_{x\ in\ Y,i}$ for a reference year *Y*. This information might be obtained using a census method to obtain adult mortality; for instance, methods that estimate the completeness of death reporting, *C*. In this case, ${}_5m_{x\ in\ Y,i}$ corresponds to the adjusted adult mortality before smoothing (Moultrie et al. 2013).
- ${}_5$Homicide rate ${}_x$ and ${}_5$Death rates in RTAs ${}_x$ for *Y* and *Y** (a year of the backward projection).
- Completeness of death reporting, *C*, for *Y* and *Y**.

Mortpak or a similar program is used to estimate the UN and Coale–Demeny model life tables. The methodology proposed by Kostaki (1991) is used to convert the 5-year life table obtained from Mortpak into a single-year table.

2.2 Method description

This section first explains how to identify and adjust a model life table to best approximate the actual life table for the reference year (steps 1–5), and then describes how to use this information to estimate the backward projection of the mortality curve (steps 6 and 7).

Part I: Identifying and modification of a model life table

Step 1: ${}_5m_x$ is estimated assuming no excess deaths due to homicides or road traffic accidents (RTAs), denoted as ${}_5m_{x\ without\ excess}$. The absence of violence is approximated by assuming that the homicide rate for men aligns with a rate that does not include excess mortality. Considering that homicide rates in the European Union (EU), North America, and globally vary between 0 and 6 homicides per 100,000 people, the *Homicide rate without excess* mortality is likely to be within this range (World Bank 2021). Similarly, the absence of excess mortality from RTAs, the *RTAs rate without excess*, could be approximated using a rate below 10, as in European countries or high-income nations. In addition, the estimation of ${}_5m_{x\ without\ excess}$ uses as inputs ${}_5$Homicide rate $_x$, ${}_5$Death rates in RTAs $_x$, and Coverage C, as the following formulas show:

$$\Delta_5\text{Homicide rate}_x = {}_5\text{Homicide rate}_x / C - \text{Homicide rate without excess}$$

$${}_5\text{Extra homicides}_x = \Delta_5\text{Homicide rate}_x \; {}_5\text{Pop}_x / 100{,}000$$

$$\Delta\, {}_5\text{Death rates in RTAs}_x = {}_5\text{Death rates in RTAs}_x / C - \text{RTA rate without excess}$$

$${}_5\text{Extra deaths in RTAs}_x = \Delta\, {}_5\text{Death rates in RTAs}_x \; {}_5\text{Pop}_x / 100{,}000$$

$${}_5\text{Total of extra deaths}_x = {}_5\text{Extra homicides}_x + {}_5\text{Extra deaths in RTAs}_x$$

$${}_5\text{Extra } m_x = {}_5\text{Total of extra deaths}_x / {}_5\text{Pop}_x$$

$${}_5\text{Deaths}_{x\text{ without excess}} = {}_5\text{Deaths}_x - {}_5\text{Total of extra deaths}_x$$

$${}_5m_{x\text{ without excess}} = {}_5\text{Deaths}_{x\text{ without excess}} / {}_5\text{Pop}_x$$

${}_5$Homicide rate $_x$ and ${}_5$Death rates in RTAs $_x$ can be obtained with Vital Statistics, and ${}_5\text{Pop}_x$ is the population.

Step 2: Identify the age x with no excess mortality using the ${}_5\text{Extra } m_x$. Model life tables must pass through ${}_5m_x$ of this point, where the curves with and without excess deaths coincide.

Step 3: Using the ${}_5m_x$ with no excess mortality identified in Step 2, the eight Coale–Demeny Model Life Tables (old and new versions of East, West, North, and South) and the ten UN Model Life Tables (old and new versions of General, Chile, Latin America,

South Asia, Far East) are estimated. These tables are represented as ${}_5m_{xF}$, where F denotes the model table. These curves can be estimated using MORPAK.
Step 4: There are two ways of identifying the model life table that better reflect the mortality pattern in the absence of excess mortality: one uses ${}_5m_x$ and ${}_5Extra\ m_x$ and the other ${}_5m_{x\ without\ excess}$; the results are the same.

- With ${}_5m_x$, the selected life table ${}_5m_{x\ selected}$ is the one with the lowest sum of squared errors using the next equation:

$$Error\ with\ F = \sum_{x=A}^{B} \left(\left({}_5m_{x\,F} + {}_5Extra\ m_x \right) - {}_5m_x \right)^2$$

- With ${}_5m_{x\ without\ excess}$, the selected life table ${}_5m_{x\ selected}$ is the one with the smallest sum of squared errors, calculated with the following equation:

$$Error\ with\ F = \sum_{x=A}^{B} \left({}_5m_{x\,F} - {}_5m_{x\ without\ excess} \right)^2$$

A and B depend on the extent of the adult mortality hump and the ages most affected by this phenomenon.

Step 5: After having ${}_5m_{x\ selected}$, the ${}_5m_{x\ estimated}$ is calculated with the following formula:

$${}_5m_{x\ estimated} = {}_5m_{x\ selected} + {}_5Extra\ m_x$$

$${}_nq_{x\ estimated} = 2\ n\ {}_nm_{x\ estimated} / (2 + n\ {}_nm_{x\ estimated})$$

Then, ${}_nq_{x\ estimated}$ is introduced into the Kostaki (1991) method to obtain ${}_1q_{x\ estimated}$.

Part II: Backward projection

Step 6: The selected life model table is estimated for each year of the backward projection Y*, ${}_5m_{x\ selected\ in\ Y*}$. The ${}_5m_x$ introduced in Mortpak or another program is the one for age x with no excess mortality, identified in Step 2. The trend over time of this ${}_5m_x$ can be obtained with Brass–Coale's indirect method or by following another method or assumption.

Step 7: Following the formulas of Step 1, we can calculate ${}_5m_{x\ estimated\ in\ Y*}$ for the year Y*. Therefore, we need the following information: ${}_5$Homicide rate ${}_{x\ in\ Y*}$, ${}_5$Death rates in RTAs ${}_{x\ in\ Y*}$, Coverage in Y*, and ${}_5m_{x\ selected\ in\ Y*}$ (obtained in Step 6). Therefore:

$${}_5m_{x\ estimated\ in\ Y*} = {}_5m_{x\ selected\ in\ Y*} + {}_5Extra\ m_{x\ in\ Y*}$$

$${}_nq_{x\ estimated\ in\ Y*} = 2\ n\ {}_nm_{x\ estimated\ in\ Y*} / (2 + n\ {}_nm_{x\ estimated\ in\ Y*})$$

Finally, ${}_nq_{x\ estimated\ in\ Y*}$ is introduced into the Kostaki (1991) method to obtain ${}_1q_{x\ estimated\ Y*}$.

3. Example or empirical training

In this section, we estimate the 1990, 1995, 2000, 2005, 2010, 2015, and 2017 male mortality curves for Colombia.

3.1 Data

The data used in the example is completely public. It was sourced from the official website of the National Administrative Department of Statistics (DANE) in January 2025 and is outlined below:

- ${}_5m_x$ for the reference year Y = 2017, the mortality reference year for the 2018 Census (DANE – Estimaciones del cambio demográfico.).
- ${}_5Pop_x$ the number of people by 5-year age group for Y and Y* (backward projection years). It is sourced from DANE's population projections (DANE – Proyecciones de población).
- Completeness of death reporting, *C*. The method used by DANE to estimate completeness using the 2018 Census and the 2017 mortality curves is outlined in Appendix 1. Taking DANE's estimates, we calculated completeness for the age range 15–64 to avoid fluctuations in the estimates at the oldest and youngest ages,

as Tools for Demographic Estimation recommends. According to Vital Statistics, in 2017 the number of deaths of men aged 15 to 64 was 47,707, and according to the $_5m_x$ estimated by DANE it was 62,193, resulting in a completeness of 0.767. We assume that this value has remained constant over the past 35 years.

- $_5$Homicide rate$_x$ and $_5$Death rate in RTAs$_x$ were estimated with the public information of Vital Statistics (https://microdatos.dane.gov.co/index.php/catalog/DEM-Microdatos) and population projections for Y and Y*.

3.2 Application of the method to Colombian males

Part I: Identifying and modifying a model life table

Step 1: Table 1 shows how $_5m_{x\ without\ excess}$ is estimated. We assume a homicide rate without excess equals 1 per 100,000, and a rate for RTAs without excess equals 5 per 100,000. These values were chosen considering the figures of the World Bank for European countries and high-income nations.

Step 2: According to $_5$Extra m_x, the age x with no excess mortality is 5 (see Table 1). For this age range, $_5$Extra m_x equals zero. At this point, the curves with and without excess deaths should coincide.

Step 3: Using the $_5m_5$ (Table 1), the Coale–Demeny and UN Model Life Tables are estimated. They are shown in Figure 4.

Step 4: First, we need to determine the values of A and B to identify the model life table that better reflects the mortality pattern in the absence of excess mortality. These values depend on the size of the adult mortality hump. Studies indicate that the exact form and timing of the hump differ across countries and evolve over time (Remund, Camarda, and Riffe 2021). In the United States the hump can reach 30 years or slightly older, and the causes are commonly senescent (Remund, Camarda, and Riffe 2018). However, the Colombian hump reaches ages beyond 30. This can be confirmed with Table 2 and Figure 3. On the one hand, Table 2 shows that the percentage of excess deaths is 61.7% for men aged 15–19 and 15.1% for men aged 50–54. On the other hand, considering $_5m_x$ and $_5m_{x\ without\ excess}$ in Figure 3, we can observe that the adult mortality hump goes approximately from age 15 to 50. Therefore, A = 15 and B = 50.

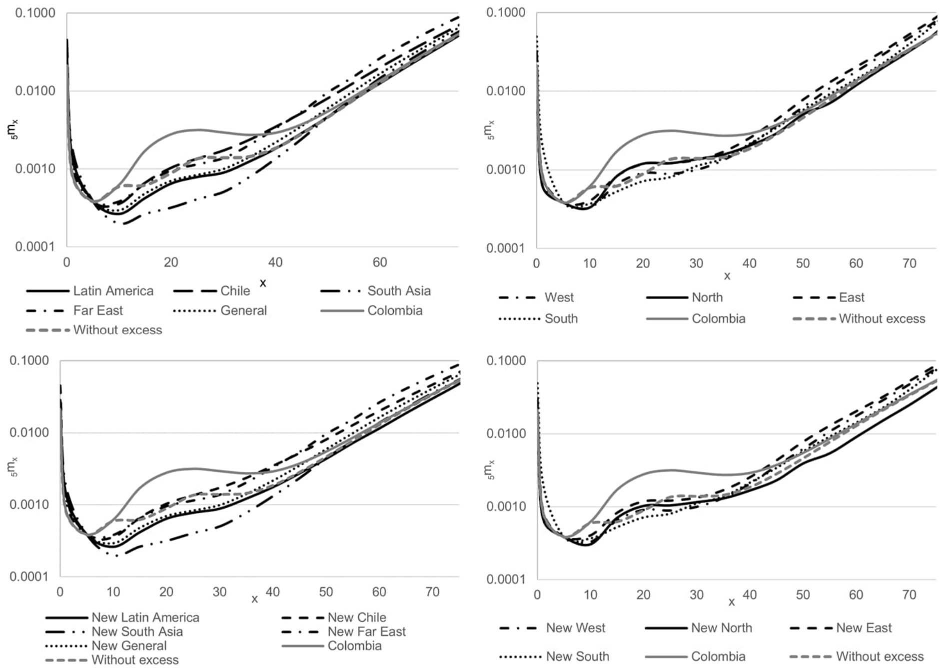
Table 1: Estimation of $_nm_x$ for males in the absence of an excess of homicides and deaths caused by RTAs, 2017

Age x	$_5Pop_x$	$_5m_x$	$_5Deaths_x$	$_5$Homicide rate $_x$	Δ $_5$Homicide rate $_x$	$_5$Extra homicides $_x$	$_5$Death rates in RTAs $_x$	Δ $_5$Death rates in RTAs $_x$	$_5$Extra deaths in RTAs $_x$	$_5$Total of extra deaths $_x$	$_5$Extra m_x	$_5Deaths_x$ without excess	$_5m_x$ without excess
0	383,542	0.02110	8,093	3.938	4.133	15.853	2.503	0.000	0.000	16	0.00004	8,077	0.02106
1	1,542,391	0.00087	1,348	1.329	0.732	11.296	3.112	0.000	0.000	11	0.00001	1,337	0.00087
5	1,955,649	0.00039	755	1.048	0.366	7.164	1.582	0.000	0.000	7	0.00000	747	0.00038
10	2,024,397	0.00062	1,260	2.718	2.543	51.479	3.899	0.083	1.671	53	0.00003	1,207	0.00060
15	2,112,841	0.00171	3,616	61.371	79.007	1669.283	24.230	26.588	561.755	2,231	0.00109	1,385	0.00066
20	2,092,635	0.00278	5,823	105.524	136.566	2857.835	39.754	46.826	979.889	3,838	0.00189	1,986	0.00095
25	1,919,713	0.00315	6,044	102.779	132.988	2552.984	35.168	40.847	784.144	3,337	0.00180	2,707	0.00141
30	1,753,083	0.00293	5,129	87.319	112.833	1978.054	31.332	35.846	628.407	2,606	0.00154	2,522	0.00144
35	1,604,728	0.00272	4,369	71.661	92.421	1483.103	28.114	31.651	507.907	1,991	0.00128	2,378	0.00148
40	1,395,988	0.00290	4,049	56.566	72.742	1015.475	25.747	28.565	398.769	1,414	0.00105	2,588	0.00189
45	1,336,171	0.00373	4,982	46.019	58.993	788.248	25.782	28.611	382.294	1,171	0.00091	2,635	0.00285
50	1,272,840	0.00547	6,960	38.986	49.824	634.184	28.741	32.468	413.269	1,047	0.00085	3,811	0.00465
55	1,095,993	0.00854	9,356	31.891	40.574	444.692	32.211	36.992	405.426	850	0.00080	5,913	0.00776
60	870,564	0.01363	11,866	25.527	32.278	280.999	33.446	38.602	336.054	617	0.00073	11,249	0.01292
65	660,459	0.02184	14,421	20.744	26.042	172.000	31.490	36.052	238.107	410	0.00064	14,011	0.02121
70	471,840	0.03456	16,307	16.690	20.758	97.944	44.756	53.346	251.709	350	0.00077	15,958	0.03382
75	312,157	0.05379	16,790	14.169	17.471	54.538	69.018	84.975	265.255	320	0.00106	16,471	0.05276
80	195,207	0.08153	15,916	14.921	18.452	36.019	68.843	84.747	165.431	201	0.00107	15,714	0.08050
85	113,289	0.11830	13,402	7.618	8.931	10.118	77.199	95.640	108.349	118	0.00108	13,283	0.11725
90	55,821	0.16286	9,091	5.798	6.558	3.661	40.124	47.308	26.408	30	0.00056	9,061	0.16232
95	21,671	0.21513	4,662	0.000	0.000	0.000	29.529	33.496	7.259	7	0.00035	4,655	0.21479
100	5,982	0.29873	1,787	0.000	0.000	0.000	0.000	0.000	0.000	0	0.00000	1,787	0.29873

Source: Public Data of Vital Statistics and population projections 2020, DANE's 2017 mortality estimates.

Note: Δ$_5$Homicide rate $_x$ = $_5$Homicide rate $_x$/ C - Homicide rate without excess, where Coverage = 0. 7671 and Homicide rate without excess =1 per-100,000 inhabitants.
$_5$Extra homicides $_x$= Δ$_5$Homicide rate$_x$ $_5Pop_x$ / 100,000
Δ $_5$Death rates in RTAs $_x$ = $_5$Death rates in RTAs $_x$ / C - RTA rate without excess, where RTA rate without excess = 5 per 100,000 inhabitants.
$_5$Extra deaths in RTAs $_x$= Δ $_5$Death rates in RTAs $_x$ $_5Pop_x$ / 100,000
$_5$Total of extra deaths $_x$=$_5$Extra homicides $_x$ + $_5$Extra deaths in RTAs $_x$
$_5$Extra m_x = $_5$Total of extra deaths $_x$ / $_5Pop_x$
$_5Deaths_{x\ without\ excess}$=$_5Deaths_x$ - $_5$Total of extra deaths $_x$
$_5m_{x\ without\ excess}$=$_5Deaths_{x\ without\ excess}$/$_5Pop_x$

Figure 4: UN and Coale–Demeny curves for Colombian males, $_5m_x$, 2017



Source: Vital Statistics 2016–2019, DANE’s 2017 mortality estimates, Coale–Demeny curves estimated with Mortpak

Table 2: Percentage of excess mortality or deaths, 2017

Age x	${}_5$Extra m_x/${}_5m_x$ x100
0	0.20%
1	0.84%
5	0.95%
10	4.22%
15	61.71%
20	65.90%
25	55.21%
30	50.82%
35	45.58%
40	34.93%
45	23.50%
50	15.05%
55	9.09%
60	5.20%
65	2.84%
70	2.14%
75	1.90%
80	1.27%
85	0.88%
90	0.33%
95	0.16%
100	0.00%

Source: Vital Statistics 2016–2019, DANE's 2017 mortality estimates.

After choosing A and B, the sum of squared errors for each model life table is estimated. The results are shown in Table 3. North obtained the lowest sum. Figure 3 shows how the North model life table and ${}_5m_{x\ without\ excess}$ overlap. Therefore, this model life table better fits the mortality pattern without excess deaths for Colombian males, becoming ${}_5m_{x\ selected}$.

Step 5: With ${}_5m_{x\ selected}$, ${}_5m_{x\ estimated}$ is calculated. The estimation is presented in Table 4. The last column is the sum of the two previous columns. Based on this table, ${}_5q_x$ is calculated for all cases, presented in Table 5. Figure 5 shows the probability of dying for single-age groups after applying Kostaki (1991) to Table 5.

Table 3: The sum of squared errors between each model life table and the curve without excess deaths

Model life table	Sum of squared errors
Latin America	0.000000885
Chile	0.0000211
South Asia	0.00000334
Far East	0.0000344
General	0.00000323
West	0.000004
North	**0.000000369**
East	0.0000129
South	0.00000227
New Latin America	0.00000104
New Chile	0.000021
New South Asia	0.00000357
New Far East	0.0000343
New General	0.0000028
New West	0.00000358
New North	0.00000107
New East	0.0000128
New South	0.00000222
Minimum	**0.000000369**

Source: Vital Statistics 2016–2019, DANE's 2017 mortality estimates, Coale–Demeny curves estimated with Mortpak.

Table 4: Calculating $_5m_x$ estimated

Age	$_5m_{x\ Colombia\ -\ 2017}$	$_5m_{x\ Without\ excess}$	$_5m_{x\ selected}$	$_5$Extra m_x	$_5m_x$ estimated
0	0.02110	0.02106	0.01560	0.00004	0.01564
1	0.00087	0.00087	0.00066	0.00001	0.00066
5	0.00039	0.00038	0.00039	0.00000	0.00039
10	0.00062	0.00060	0.00033	0.00003	0.00036
15	0.00171	0.00066	0.00083	0.00106	0.00188
20	0.00278	0.00095	0.00120	0.00183	0.00303
25	0.00315	0.00141	0.00122	0.00174	0.00296
30	0.00293	0.00144	0.00137	0.00149	0.00286
35	0.00272	0.00148	0.00157	0.00124	0.00281
40	0.00290	0.00189	0.00205	0.00101	0.00306
45	0.00373	0.00285	0.00291	0.00088	0.00379
50	0.00547	0.00465	0.00509	0.00082	0.00591
55	0.00854	0.00776	0.00708	0.00078	0.00786
60	0.01363	0.01292	0.01200	0.00071	0.01271
65	0.02184	0.02121	0.02000	0.00062	0.02062
70	0.03456	0.03382	0.03270	0.00074	0.03344
75	0.05379	0.05276	0.05620	0.00102	0.05722
80	0.08153	0.08050	0.13240	0.00103	0.13343
85	0.11830	0.11725		0.00105	
90	0.16286	0.16232		0.00054	
95	0.21513	0.21479		0.00034	
100	0.29873	0.29873		0.00000	

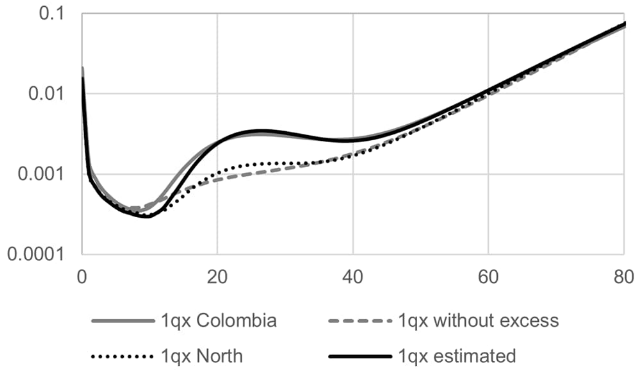
Source: Vital Statistics 2016–2019, DANE’s 2017 mortality estimates, Coale–Demeny curves estimated with Mortpak.

Table 5: Probabilities of dying, $_5q_x$

Age	$_5q_x$ - Colombia 2017	Without excess	$_5q_x$ - selected	$_5q_x$ estimated
0	0.020880	0.020839	0.015479	0.015520
1	0.003491	0.003461	0.002625	0.002654
5	0.001927	0.001909	0.001928	0.001946
10	0.003108	0.002977	0.001669	0.001800
15	0.008520	0.003271	0.004136	0.009381
20	0.013818	0.004733	0.005982	0.015055
25	0.015619	0.007026	0.006082	0.014683
30	0.014522	0.007168	0.006827	0.014183
35	0.013520	0.007381	0.007819	0.013956
40	0.014398	0.009393	0.010198	0.015199
45	0.018470	0.014161	0.014445	0.018753
50	0.026972	0.022960	0.025130	0.029134
55	0.041791	0.038066	0.034784	0.038522
60	0.065903	0.062583	0.058252	0.061587
65	0.103526	0.100731	0.095238	0.098050
70	0.159063	0.155919	0.151144	0.154305
75	0.237063	0.233074	0.246383	0.250312
80	0.338639	0.335071	0.497367	0.500274
85	0.456483	0.453363		
90	0.578688	0.577326		
95	0.699454	0.698745		
100	0.855066	0.855066		

Source: Vital Statistics 2016–2019, DANE's 2017 mortality estimates, Coale–Demeny curves estimated with Mortpak.

Figure 5: Comparison of the probabilities of dying, $_1q_x$



Notes: Kotaski (1991) was used to approximate the curves to single ages.
Source: Vital Statistics 2016–2019, DANE's 2017 mortality estimates, and Coale–Demeny curves estimated with Mortpak.

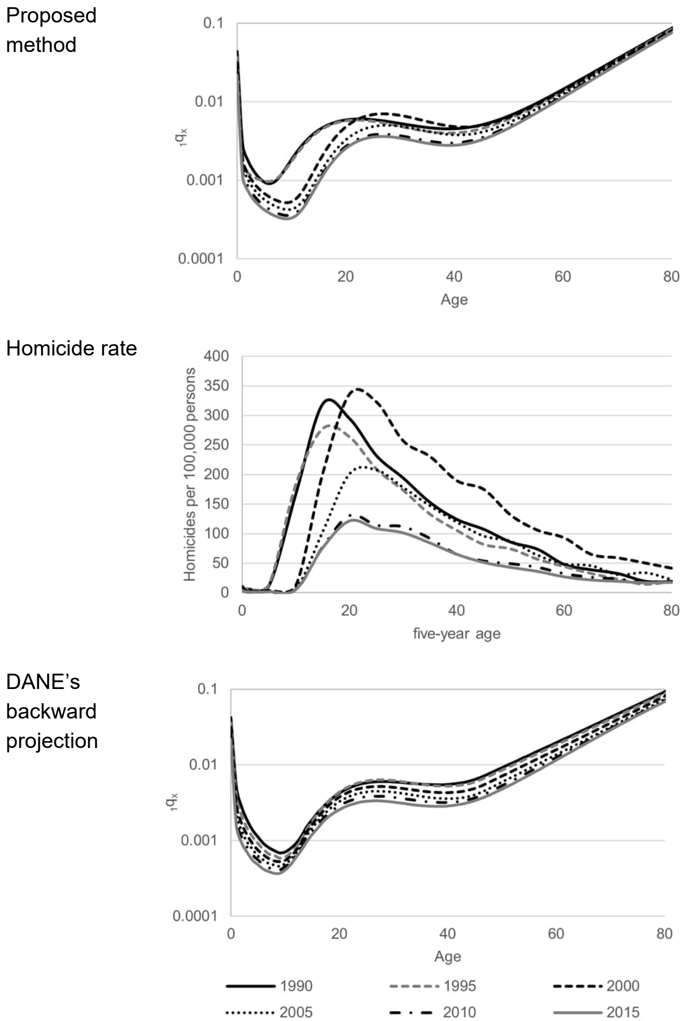
Part II: Backward projection

Step 6: The mortality curves, $_5m_{x\ selected}$, are estimated for 1990, 1995, 2000, 2010, and 2015 using the North model life tables and the $_5m_5$ (identified in Step 2) estimated for these years. As explained, $_5m_5$ can be indirectly estimated using Brass–Coale or following another method or assumption. We decided to use the values estimated by DANE to make both backward projections comparable.

Step 7: In this step, the formulas of Step 1 are used to calculate $_5m_{x\ estimated\ in\ Y*}$ for Y* equals 1990, 1995, 2000, 2010, and 2015. The basic inputs are the curves estimated in Step 6 and the 5-year age group homicide and RTA death rates for men for these years, which allow us to calculate $_5Extra\ m_{x\ in\ Y*}$. $_5q_{x\ estimated\ in\ Y*}$ is calculated based on $_5m_{x\ estimated\ in\ Y*}$ and then is approximated to single ages with Kotaski (1991). The upper part of Figure 6 shows the $_1q_{x\ estimated\ in\ Y*}$. Figure 6 compares the curves $_1q_x$ for 1990, 1995, 2000, 2010, and 2015 ($_1q_{x\ estimated\ in\ Y*}$), obtained with the proposed method, to the 5-year age group homicide rates and the DANE $_1q_x$ curves for these years.

Figure 6 shows that the $_1q_{x\ estimated\ in\ Y*}$ curves reflect the homicide rate patterns for the 5 years. This is mainly evident due to the differing homicide patterns between the first two years and the last three. In 1990 and 1995 the highest homicide rates were for men aged 15 to 19, while in 2000, 2010, and 2015 they were highest for those aged 20 to 24. This change in pattern is reflected in the gap between the 1995 and 2000 curves for ages 10 to 20, as observed in the upper graph. It is important to clarify that the gap could have been smaller if we had used the single-age group homicide rate; however, this information is not publicly available.

Figure 6: Backward projection of male mortality curves and historical homicide rate for every 5 years from 1990 to 2015



Notes: Kotaski (1991) was used to approximate the curves to single ages.
Source: Vital Statistics 2016–2019, DANE's 2017 mortality estimates, Coale–Demeny curves estimated with Mortpak.

The probabilities of dying estimated with the proposed method also show that the year 2000 had the highest homicide rates for ages above 20 (which is observed in the homicide rate graph), in contrast to the DANE curves. Therefore, ${}_1q_{x\ estimated\ in\ 2000}$ surpasses the curves ${}_1q_{x\ estimated\ in\ 1995}$ and ${}_1q_{x\ estimated\ in\ 1990}$ by 20 to 45 years. By contrast, the DANE curve for 2000 is below those of 1995 and 1990 for all ages.

4. Discussion

Table 6 compares the life-expectancy-at-birth estimates of the probability-of-dying curves of Table 5. On the one hand, the difference of e(0) between DANE and the estimate with the proposed method is 0.30 years, and the difference between without excess and North is 0.34. On the other hand, $\Delta e(0)_{\text{no-excess and excess}}$ in 2017 is about –2.4 years using both DANE information and the results of the proposed method. This value is close to 2.43 AYLL estimated with Arriaga's method (DANE 2022a).

Table 6: Comparison of life expectancy at birth

Curve	e(0) in 2017
Colombia 2017 (DANE)	72.72
Without excess	75.15
Estimated with the proposed method	72.42
Selected: North	74.81
	Δ e(0) 2017
DANE, Without excess	–2.43
Estimated with the proposed method, North	–2.39

Note: To make comparable the life-expectancy-at-birth estimates of the probability-of-dying curves of, we considered the data of ages 0 to 75 years and an infant mortality of 0.0208 for all scenarios.
Source: Vital Statistics 2016–2019, DANE 2017 mortality estimates, Coale–Demeny curves estimated with Mortpak.

Table 7 compares life expectancy at birth with and without excess mortality for the years in the backward projection, $\Delta e(0)_{\text{no-excess and excess}}$, which were estimated with the proposed method. Colombia has experienced significant violence driven by multiple factors. Although the armed conflict is often regarded as the main cause, that and other factors have intertwined and evolved in various ways, amplifying the violence throughout the country's history.

In the 1990s, a war involving drug cartels spread across the nation, along with clashes between guerrillas and the military. This helps explain the approximately 5-year decline in life expectancy at birth, $\Delta e(0)_{\text{no-excess and excess}}$, in 1990 and 1995, shown in Table 7. By the end of 1986, a group of drug traffickers, the Extraditables, led by Pablo Escobar, declared war on the Colombian state in opposition to President Virgilio Barco's restoration of extradition. By 1990 the Extraditables were responsible for 623 attacks, leaving 1,710 wounded and 550 police officers killed. Insecurity and fear grew, contributing to the sense that the state was losing the war. By the mid-1990s the dynamics of the conflict had shifted. Communities were being displaced by paramilitaries, who had formed a national project under the name of the United Self-Defence Forces of Colombia (AUC), supported by drug trafficking, a significant sector of the military, and political and economic elites. Meanwhile, the FARC-EP guerrillas unsuccessfully attempted to shift the conflict to a politically motivated war, also fuelled by coca resources. The conflict escalated into a struggle for control of territory and the population, reaching the highest levels of violence in the history of the conflict (Comisión de la Verdad 2022a).

Table 7: Comparison of life expectancy at birth for the years in the backward projection

Year	e(0) Without excess	e(0) With excess	e(0) With excess-e(0) Without excess
1990	69.67	64.80	–4.87
1995	70.97	66.07	–4.91
2000	72.07	66.60	–5.47
2005	72.96	69.45	–3.52
2010	73.78	71.17	–2.61
2015	74.47	72.00	–2.47

Note: To estimate life expectancy at birth for the years in the backward projection, we considered the data for ages 0–75 and kept DANE infant mortality rates.
Source: Vital Statistics 2016–2019, DANE mortality estimates, Coale-Demeny curves estimated with Mortpak.

The –5.47-year change in e(0), $\Delta e(0)_{\text{no-excess and excess}}$, in 2000 (Table 7) is explained mainly by the demilitarized zone of El Caguán, a territory whose objective was to boost the peace process with the FARC guerrillas and end the armed conflict. This area existed between January 1999 and February 2002. According to the Comisión de la Verdad (2022b), during the first two years of peace talks with the FARC-EP, both the government and the guerrillas maintained a dual approach, preparing for military victory while engaging in negotiations. The United Self-Defence Forces of Colombia (AUC) expanded across Colombia, targeting civilians, particularly human rights defenders and journalists, in a strategy that escalated from selective to indiscriminate violence. This led to massacres, a massive exodus to cities, and a humanitarian crisis. Meanwhile, the FARC-

EP also caused deaths and forced displacement, driven by territorial control and revenue needs.

The government of Álvaro Uribe Vélez, from 2002 to 2010, focused all its efforts on militarily regaining control of the territories held by the guerrillas, while negotiating with the AUC. During this first decade of the millennium, violence decreased, although there was a spike in 2007 due to extrajudicial executions (Comisión de la Verdad 2022b). As a result, the $\Delta e(0)_{\text{no-excess and excess}}$ went from –5.47 years in 2000 to –3.52 in 2005 and –2.61 in 2010 (Table 7).

Juan Manuel Santos won the elections in 2010 and 2014. Between 2012 and 2016, military action was combined with a political negotiation process that led to the signing of the Peace Agreement with the FARC-EP. During these years the levels of violence were the same as at the end of the previous administration (shown in Figure 3), which is reflected in the –2.47-year $\Delta e(0)_{\text{no-excess and excess}}$ in 2015 (see Table 7).

5. Conclusion

Although life table models have become less relevant in recent decades, this study demonstrates that they can still be useful if their limitations are considered. This article enriches the scientific literature by presenting a simple and intuitive method that modifies the Coale–Demeny and UN model life tables to approximate mortality curves for populations with excess mortality due to external causes. It systematically outlines and illustrates the method for adapting the Coale–Demeny and UN model life tables to account for the excess mortality from violence and road traffic accidents (RTAs) in Colombia. It then employs this method to estimate the backward projection of males' probability of dying at age x from 1990 to 2015 at the national level. The results are then analysed and linked to key historical events in Colombia.

The method identifies the model life table that best represents mortality patterns in the absence of excess deaths from external causes. This is a key contribution of the approach, as it enables the generation of backward projections of mortality curves in contexts where historical cause-specific mortality rates for non-external causes are unavailable. We found that the North model life table most closely aligns with the mortality pattern of Colombian males in the absence of excess deaths. By applying the backward projection of this model, along with historical male homicides and deaths due to RTAs from 1990 to 2015, we were able to estimate the reduction in life expectancy at birth of both scenarios, without and with excess mortality. Our findings reveal that RTAs and, more significantly, the wars in Colombia during the 1990s and 2000s reduced male life expectancy, measured as $\Delta e(0)_{\text{no-excess and excess}}$, by 3 to 5 years.

Finally, the effectiveness of the methodology relies on the quality of the input data; for example, how accurately the subnational mortality curves, ${}_5m_{x\ in\ \mathrm{Y}}$, are estimated for the reference year Y. In addition, as mentioned in the discussion, the estimates would have been better if we had had single-age-group homicide and RTAs death rates; however, this information is not publicly available. It is also important to note that the method depends on the availability of multiple data sources (e.g., census, vital statistics), which may be challenging to obtain in many countries where they are incomplete, inconsistent, or simply non-existent.

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Appendix

Estimation of male mortality for Colombia in 2017

(Based on DANE (2022e) Estimación de las curvas de mortalidad a nivel subnacional: Colombia 2017. Metodologías Demográficas Aplicadas, Numero 1.)

The estimation of male mortality in Colombia in 2017 used a bottom-up methodology. First, the subnational curves were estimated at the urban and rural levels, and then these curves were aggregated to estimate the national curve. This section outlines the methodology used to estimate the 2017 mortality curves at the departmental level, organized into three parts. The first part explains the application of the under-coverage factor from Vital Statistics to estimate urban life tables. The second part details the smoothing techniques employed to define the mortality pattern. The third explains how the urban curves were used to estimate the rural curves.

Estimation of urban-department mortality curves

The first step involved proportionally distributing deaths from 2016, 2017, and 2018 that lacked information on sex, department, and age. Then the average number of deaths over the three years was calculated to estimate the number of deaths for 2017. The under-coverage correction factors were derived from question 26 of the 2018 Census, which asked: Did any household member die in 2017? If the answer was yes, additional details regarding the sex, age, and the issuance of the death certificate were requested. The correction factor was calculated as the inverse of the proportion of certified deaths. In 2005 this factor was computed using the formula 1 + (deaths without certificate) / (deaths without certificate + deaths with certificate). This formula produces values that closely approximate the inverse of the proportion of certified deaths when that proportion is below 0.2. The corrected death count was determined by age, sex, and department by multiplying the death figures from Vital Statistics by the under-coverage correction factor. Later, the estimates of ${}_1m_x$ were smoothed using the cumulative rate method outlined by Arriaga (2011).

Smoothing and urban mortality patterns

The second step involved smoothing and estimating the mortality pattern after determining the life table with the corrected death counts. Two methods were used: the Age Pattern of Mortality introduced by L. Heligman and H. J. Pollard in 1980, and the

Flexible Bayesian Model for Estimating Subnational Mortality proposed by Alexander et al. (2017). The method chosen for each department was the one that minimised the sum of squared errors.

Estimation of rural-department mortality curves

Due to underreporting in rural areas regarding the death certificate question in the 2018 Census, which resulted in lower mortality rates in rural than urban areas, the method used for urban areas was not applied. Instead, the following procedure was implemented: First, logistic models for the probability of dying in 2017 were developed by merging the data of living and deceased individuals from the 2018 Census. This approach enabled the calculation of the factor ${}_1q_x$(rural) / ${}_1q_x$(urban) by gender and department. Then this factor was applied to estimate ${}_1q_x$(rural) by adjusting the probability of dying in rural areas based on the urban mortality rates, calculated previously.