DEMOGRAPHIC RESEARCH
A peer-reviewed, open-access journal of population sciences

DEMOGRAPHIC RESEARCH

VOLUME 54, ARTICLE 12, PAGES 371–404
PUBLISHED 3 MARCH 2026
https://www.demographic-research.org/Volumes/Vol54/12/
DOI: 10.4054/DemRes.2026.54.12

Research Article

A flexible model for reconstructing education-specific fertility rates: The case of sub-Saharan Africa

Dilek Yildiz

Arkadiusz Wiśniowski

Zuzanna Brzozowska

Afua Durowaa-Boateng

Contents

Demographic Research: Volume 54, Article 12
Research Article

A flexible model for reconstructing education-specific fertility rates: The case of sub-Saharan Africa

Dilek Yildiz[1]

Arkadiusz Wiśniowski[2]

Zuzanna Brzozowska[3]

Afua Durowaa-Boateng[4]

Abstract

BACKGROUND
Accurate and harmonized estimates of education-specific fertility rates are crucial for understanding the past and projecting the future human population. Yet fertility estimates derived from demographic surveys that collect detailed fertility histories often do not align with the reliable and widely used United Nations (UN) World Population Prospects. This inconsistency means that the choice of data source can affect research outcomes on fertility trends.

OBJECTIVE
We combine the patchy Demographic and Health Surveys (DHS) data and the UN total fertility rate (TFR) estimates to create three harmonized datasets of education-specific TFRs for 36 sub-Saharan African countries, with different degrees of consistency with the UN TFR.

METHODS
We develop a flexible Bayesian hierarchical model that reconstructs education-specific fertility rates by combining the DHS data and the UN TFR estimates.

[1] International Institute for Applied Systems Analysis, Wittgenstein Centre for Demography and Global Human Capital (IIASA, OeAW, University of Vienna), Laxenburg, Austria. Email: yildiz@iiasa.ac.at.
[2] Social Statistics Department, The University of Manchester, United Kingdom.
[3] Austrian National Public Health Institute (Gesundheit Österreich GmbH, GÖG).
[4] Austrian Academy of Sciences Vienna Institute of Demography, Wittgenstein Centre for Demography and Global Human Capital (IIASA, OeAW, University of Vienna), Vienna, Austria.

RESULTS
We provide time series of education-specific TFR quinquennial estimates between 1980 and 2014 for 36 sub-Saharan African countries. We present three model specifications that provide the users with fertility estimates that differ in their degree of consistency with the UN TFR.

CONCLUSIONS
The model estimates show significant variation across countries, leading to divergent fertility trends from 1980 to 2014, mainly in levels but sometimes also in direction.

CONTRIBUTION
Our flexible modelling framework offers model specifications that suit different conditions and can obtain results consistent with stakeholder needs: consistent with but not identical to the UN, fully consistent (nearly identical) with the UN, and consistent with the DHS. Further, our estimates of education-specific TFR can be used to analyse and forecast fertility trends and their contribution to population change in 36 sub-Saharan African countries.

1. Introduction

In this article, we propose a flexible Bayesian hierarchical model to reconstruct fertility rates for four educational categories by using the full birth history module from the Demographic and Health Surveys (DHS; USAID 2023) and the TFR estimates from the United Nations World Population Prospects (UN WPP; UN 2022). Our resulting estimates cover 5-year periods between 1980 and 2014 and fill the gap in the time series for 36 sub-Saharan African countries. Our model is flexible, in the sense that it produces estimates of model specifications. We provide results from three models with varying levels of consistency of the resulting TFR with the UN data: (1) estimated TFR consistent with but not identical to the TFR estimated by the UN (hereafter 'main model UN-consistent') (2) estimated TFR fully consistent with (nearly identical to) the TFR estimated by the UN (hereafter 'UN-fully consistent'), and (3) estimated TFR consistent only with the TFR estimated by the DHS (hereafter 'DHS-consistent').

We consider the estimates from the main model as the first choice for researchers and policymakers because they average the UN and the DHS data when there is a large discrepancy between the two sources. When the UN and the DHS data are close to each other, this is preserved in the final model estimate. The UN-fully consistent and the DHS-consistent models would be a good choice for researchers and policymakers who are using multiple indicators from these sources and need consistency while using the complete time series in their analysis. Additionally, using all three sets of estimates

enables researchers to conduct sensitivity analysis, testing the robustness of their results with different sets of fertility estimates.

This work is related to the estimates developed by Durowaa-Boateng, Yildiz, and Goujon (2023), who reconstruct age- and education-specific fertility rates that are consistent with the UN fertility rates in a Bayesian framework. They employ as a benchmark one of the three sets of estimates (the 'UN-consistent' estimates) constructed using an earlier version of the methods developed in this article (available in Yildiz et al. 2023). However, model specifications in this article differ from those in Durowaa-Boateng, Yildiz, and Goujon (2023), which constitutes a critical contribution to the literature. We discuss these differences below.

This study contributes to the literature in multiple ways that distinguish it from Durowaa-Boateng, Yildiz, and Goujon (2023), especially in the use of the DHS estimates and how potential biases in DHS data are accounted for. First, whereas Durowaa-Boateng, Yildiz, and Goujon (2023) derive education-specific TFRs (ESTFRs) indirectly from the age-specific fertility rates, in our modelling framework we estimate the ESTFRs directly. This shift allows more precise alignment with the UN and DHS TFRs. Second, this article expands on the modelling framework to build three models with different levels of consistency with the UN and the DHS TFR estimates, in comparison to only one model which is consistent with the UN age-specific fertility rate (ASFR). This demonstrates the flexibility of the statistical modelling framework, which enables results consistent with stakeholders' needs. Third, our methodology is enhanced by the use of several measurement error models that account for the quality of the surveys. This is an important improvement over Durowaa-Boateng, Yildiz, and Goujon (2023), who employ a Generalized Linear Model and only partially correct for survey quality. Finally, our model borrows information on sub-regions in Africa through prior distributions, while Durowaa-Boateng, Yildiz, and Goujon (2023) do not take that into consideration in their modelling. These differences, when considered as a whole, demonstrate the flexibility and robustness of our methodological framework, which meets the estimation and benchmark requirements of different users.

The remainder of the article is as follows. In the next section we provide a brief background to the relationship between educational attainment and fertility levels and existing approaches to modelling fertility in a context of scarce and inconsistent data. Next, we present the data sources and our methodology. In Section 4 we assess the estimates from the three models against the DHS and UN data. In the final section we evaluate the education-specific estimates of fertility trends between 1980 and 2014 generated by the models. We also provide detailed documentation of the proposed method so that it can be applied to other countries, regions, and time periods.

2. Background

The future world population growth and size will be largely determined by the pace of fertility decline in sub-Saharan Africa (Bongaarts 2006, 2008; Shapiro and Gebreselassie 2008). A key factor will be women's educational attainment (Lutz et al. 2018), because global trends show that fertility has a strong and negative correlation with female education: Women with higher levels of education usually have fewer children than those with lower levels of education (Amin and Behrman, 2011; Bongaarts 2010; Castro Martin 1995; Gebreselassie and Shapiro 2016).[5] This link is particularly strong in low-income and high-fertility regions such as sub-Saharan Africa (Skirbekk 2008). Kebede, Goujon, and Lutz (2019) have shown that the fertility-decline stalls[6] observed in some sub-Saharan African countries during the late 1990s and early 2000s resulted, at least partly, from disruptions in female educational expansion in the 1980s. In a recent decomposition study, involving more data and a different definition of fertility stalls, Schoumaker and Sanchez (2024) reported that while the slowdown in educational progress was not the main reason for the stalls in the fertility decline in sub-Saharan African countries, it made certain countries more vulnerable to stalls.

The mechanisms that link education – especially higher education – to lower fertility levels are numerous. First, education provides women with autonomy, allowing them knowledge and use of contraception (Bongaarts 2010). Also, women who dedicate time to their education usually put off marriage and childbearing until after their education is completed, so that women with higher education delay childbirth (Schultz 1997). Furthermore, women with higher education and women who live in urban areas tend to have more opportunities to enter the labour market and earn higher wages, thereby creating an opportunity cost of childbearing (May and Rotenberg 2020). The effect of education expansion and rapid urbanisation is visible in successive birth cohorts, especially among women with secondary or higher schooling and those residing in urban areas, who tend to start childbearing later (postponement) and subsequently have fewer births. Period-specific declines are pronounced at younger ages, with a relative shift toward older ages. Considering these tempo (timing) and quantum (level) differences helps to interpret changes in period total fertility rates, which are the focus of this study (Ryder 1965; Bongaarts and Feeney 1998; Bongaarts 2017).

Even though the literature shows a mostly negative relationship between education and fertility levels, even in high fertility settings this relationship is not always true. In

[5] In some low-fertility countries, this relationship has recently changed into a positive one or an inverted U-shape pattern where women with medium education have higher fertility than their peers with the highest and lowest levels of education (Jalovaara et al. 2019). However, the current paper focuses on high-fertility countries, where the association remains strictly negative.

[6] According to Bongaarts (2008: 109), "a stall implies that an ongoing fertility transition is interrupted by a period of no significant change in fertility before the country reaches the end of the transition".

many low- and middle-income countries where fertility is high, before the demographic transition, women with some educational attainment (e.g., primary education) had an initial higher fertility level than women with no education (Martin 1995). However, at the onset of the demographic transition, the negative relationship between education and fertility levels resumed (Shapiro 2012).

The importance of education for fertility is also reflected in multi-state projections, where education-specific fertility rates are among the key input data. Population projections that account for an educational component have been shown to consistently result in lower future population counts than those that exclude education (Gietel-Basten and Sobotka 2021; Lutz and KC 2011). Yet consistent, cross-country, comparable estimates of education-specific fertility – in particular total fertility rates (TFR) for sub-Saharan African countries – remain scarce.

The Demographic and Health Surveys (USAID 2023), which were launched in 1984, are the main source of comparative demographic data for sub-Saharan Africa. They are a series of repeated cross-sectional representative surveys conducted in over 90 low- and middle-income countries. The DHS data are usually collected at 5-year intervals and provide detailed information about respondents' socioeconomic characteristics, infant and child mortality, and women's birth histories. They are one of the main data sources used in the United Nations (UN) population projections for low- and middle-income countries (UN 2022) and are the only data source that allows for comparative studies of the education–fertility relationship in these countries (Kebede, Goujon, and Lutz 2019; Schoumaker 2008; Sneeringer 2009).

As indispensable as the DHS data are for researchers, policymakers, planners, and international organisations, they have serious quality issues. First, most of the African countries included in the DHS have only conducted a few nationally representative fertility surveys, of varying quality, since the 1980s, causing missingness and inconsistencies in the time series (Schoumaker 2014). The surveys cover different periods in different countries, and the obtained data series have temporal gaps. Second, the data demonstrate the typical limitations of sample surveys: sparse observations for some population groups, measurement errors (e.g., misreporting), and sampling errors. For instance, the share of women with high education is so low in some low-income countries that the values obtained through surveys are insufficient to make reliable fertility rate estimations. Furthermore, measurement errors may concern the timing of the reporting (e.g., heaping birthdates on years ending with 0 or 5 and the Potter effect, which happens when the timing of distant births is reported as being closer to the survey date and causes underestimation of distant fertility rates and overestimation of fertility in intermediate periods) and/or the number of events (i.e., omission of births), and typically increase in parallel with the timespan between the estimated period and the time of data collection (the DHS data on birth histories allow for estimating retrospective fertility rates up to 25

years prior to the survey date). Sampling errors usually include the over- or under-sampling of certain population groups, as well as selection bias caused by selective mortality and/or migration (Schoumaker 2014).

These quality issues result in inconsistencies in estimates across sources, countries, and time (Al Zalak and Goujon 2017; Schoumaker 2014, 2011), and particularly in biased estimates of period fertility and mortality rates (Alkema et al. 2012; Rajaratnam et al. 2010; Schoumaker 2013). Recently, it has been shown that the aforementioned stalls in fertility decline in sub-Saharan Africa may be much less widespread than the published DHS data suggest (Schoumaker 2019). Schoumaker has observed that "taking published [DHS] fertility figures at face value could be risky in some contexts", whereby "[i]nferring fertility trends by comparing recently published [DHS] fertility data from successive surveys may lead to erroneous trend results" (Schoumaker 2014: xi). He therefore corrected the reconstructed fertility rates from DHS birth histories using a Poisson regression (Schoumaker 2013, 2010). Although his model can be enhanced by education parameters to estimate fertility by education level, the resulting fertility estimates are higher than those published by the UN Population Division, the main provider of UN global population estimates and projections. This is because the fertility rates reported in the DHS are often higher than the UN fertility rates.

The UN is aware of the DHS quality issues and therefore does not exclusively rely on it for producing the population estimates and projection datasets published in the World Population Prospects (WPP). Rather, the UN combines DHS data with other data sources and regularly updates its estimates of past fertility rates along with other demographic rates and population counts. It uses an iterative process to improve past fertility rate estimates and ensure their consistency with other demographic components and over time. Furthermore, the UN WPP 2022 revisions for 1950–2100 fertility estimates follow and build on the methodologies of Liu and Raftery (2020), Ševčíková, Alkema, and Raftery (2011), and Alkema et al. (2012). All these methodologies consider transition periods, biases, and errors in data sources. Specifically, in Liu and Raftery (2020), phases of fertility transition are modelled in a four-level Bayesian hierarchical model, which consists of three levels for each of the transition periods (pre-fertility transition, transition, and post-transition) and one level for data sources. TFRs in pre-fertility transition phases and fertility transition phases are modelled by random walk models, while TFRs in the post-transition phase follow an autoregressive model. Bias and the measurement error variance of the observed TFRs are estimated by the difference between the observed and the UN TFRs and by regressing error terms on the data-quality indicators. The methodologies of the UN WPPs use previous revisions as benchmark values and further build upon these past estimates. More specifically, Alkema et al. (2012) develop a probabilistic model to estimate the trends in TFR and their uncertainty for several West African countries by decomposing the measurement error into bias and

variance. However, these estimates are only available for the overall fertility rates and not by the level of educational attainment.

3. Data and methods

3.1 Data

We used two sources of fertility data: the UN World Population Prospects (WPP, UN 2022) and surveys conducted under the umbrella of the Demographic and Health Surveys (DHS). The UN WPP 2022 total fertility rates (hereafter UN TFR) are estimates based on multiple data sources and estimation methods (UN 2022). Given that the UN TFR estimates are derived from comprehensively validated probabilistic models that account for biases and temporal transitions, they provide a consistent and reliable basis for the analysis presented in the following section (Liu and Raftery 2020; Ševčíková, Alkema, and Raftery 2011). These UN TFR estimates, which we treated as data inputs, were rigorously checked for consistency within and across countries and they were consistent with the other UN WPP rates, population estimates, and projections. The UN TFR estimates are updated approximately every two years as new data sources become available. Although they might be subject to sampling and modelling errors as well as biases, like Alkema et al. (2012: 340), we did not assume that these "errors are not consistently either too high or too low on average". The methodology employed by the UN WPP 2022 revisions takes into account biases and errors in their input data (United Nations, Department of Economic and Social Affairs, Population Division 2022). In addition, although not without flaws, the UN WPP are widely considered the standard and are adopted by many international organisations (Liu and Raftery 2020; Lutz and KC 2010).

The DHS survey data includes 178 datasets from 36 sub-Saharan African countries, of which 134 are Standard DHS surveys, 30 are Malaria Indicator Surveys (MIS), 9 are Continuous DHS surveys, 4 are Standard AIDS Indicator Surveys (AIS), and 1 is an Interim DHS survey. These combined surveys yield a sample of 1,684,458 women, of whom 38% (640,462) have no education and 33.2% (559,362), 25.4% (428,251), and 3.4% (56,383) have primary, secondary, and higher education, respectively. Each educational attainment level comprises 766 data points for different years spanning 1980 to 2014, thus producing a total of 3,064 data points. All survey files were downloaded from the DHS Program website (USAID 2023). Survey years and characteristics are available in Appendix A Figure A-1.

The retrospective estimates of 5-year period TFRs and education-specific, 5-year period TFRs (ESTFR) were calculated using Stata's *tfr2* module for 5-year periods from

1980–1984 to 2010–2014 (Schoumaker, 2013). The estimated rates were based on the birth histories of female survey participants aged 15 to 49. Our framework used estimates for periods 0–4 years, 5–9 years, 10–14 years, and 15–19 years prior to the survey. This ensured we had more data points than only the fertility estimates at the time of the survey. This retrospective information was especially useful for countries that conducted only one or two DHS surveys. The TFR and ESTFR estimates for 0 to 4 years before the survey are denoted by TFR0 and ESTFR0, estimates for 5 to 9 years before the survey are denoted by TFR5 and ESTFR5, and so on. Countries that conducted more than one DHS survey between 1980 and 2014 generated multiple TFR and ESTFR estimates that refer to the same 5-year period. These values formed an input to the statistical model and were assumed to be subject to bias and missingness. The biases are composite measures of misreporting in a survey (e.g., due to long recollection periods), idiosyncratic errors, and potential model misspecifications implemented in the *tfr2* Stata module. Another source of potential bias is the imputation of missing dates in birth century–month codes. The DHS program applies standardized editing and imputation during central processing when the month or year is missing or inconsistent (ICF 2018). Since our focus is on 5-year periods, we expect low levels of such imputation effects on the estimates. Our modelling framework accounts for biases for each recall period and quality group from the survey and aims to reduce the bias in our final estimates and thus produce consistent estimates over time. We used the survey quality assessments by Schoumaker (2014), who groups countries according to regularity in the case of only one survey, and consistency of fertility trends across surveys in the case of multiple surveys.

3.2 Methods: Education-specific fertility rate reconstruction model

The 5-year period total fertility rates by educational attainment were reconstructed from the 1980–1984 period to the 2010–2014 period using a hierarchical time series Bayesian model. Bayesian inference offers flexibility in integrating data of varying quality and from various sources, such as those employed here, and produces estimates of the quantity of interest with measures of uncertainty. In our model, the key assumption that we made is that both DHS and UN TFR measure the same quantity (i.e., fertility rates), but the measurement is subject to different measurement errors. These errors include potential bias (systematic error) and uncertainty (a non-systematic error, quantified in our model in terms of precision, i.e., inverse variance).

The modelling framework is depicted in Figure 1, while the model's three levels are explained below. The key output from the model is the estimated ESTFRs, alongside measures of uncertainty. These ESTFRs refer to estimates that are corrected for data

inadequacies (see Introduction) and are based on information from both the DHS and UN TFR.

The first level starts by calculating the country-specific TFRs for each 5-year period as the weighted sum of ESTFRs (Equation 1). Calculating TFRs allows us to control the consistency of our ESTFR estimates with the UN TFR. Weights $w[c,e,y]$ for each education level are equal to the proportions of women aged between 15 to 49, with educational attainment level e for a given country c and year y derived from population sizes by age and educational attainment (WIC 2018).

$$TFR[c,y] = \sum_{e=1}^{4} w[c,e,y]\, ESTFR[c,e,y], \text{ y = 1980–1984, 1985–1989, …, 2010–2014.} \quad (1)$$

Next, our TFR estimates are adjusted to the 2022 UN TFR data, denoted by $TFR_{UN}[c,y]$, as shown in Equation 2:

$$TFR_{UN}[c,y] \sim Normal(TFR[c,y], \sigma_{UN}[c,y]). \quad (2)$$

The parameter for standard deviation, $\sigma_{UN}[c,y]$, captures the uncertainty of the UN TFR. The specification of $\sigma_{UN}[c,y]$ is flexible in that it can either be set as a model parameter that is estimated from the data, or it can be fixed at a value that reflects the researcher's degree of belief in the UN TFR data. For example, setting $\sigma_{UN}[c,y] = 0.1$ would mean that the true TFR lies within the TFR_{UN} ±0.2 interval with probability 0.95.

At level 2 (Figure 1), we assume a hierarchical time series model for our key quantity of interest: a 'true' and unobserved ESTFR. The time series model assumes that each country-specific *ESTFR* follows its own autoregressive process and is modelled using a normal distribution truncated below at zero, as shown in Equation 3 and Equation 4:

$$ESTFR[c,e,y] \sim Normal(\beta_0[1,r,e], \sigma_0) \text{ where } y = 1980–1984, \quad (3)$$

$$ESTFR[c,e,y] \sim Normal(\beta_1[1,r,e] + \beta_2[1,r,e] * ESTFR[c,e,y-5], \sigma_1),$$
where y = 1985–1989, 1990–1994, ... , 2010–2014. (4)

The studied countries are divided into four UN subregions (subscript r): Eastern, Middle, Southern, and Western Africa (see Table A-1). Countries in the same subregion r have the same prior distributions for β_0 and β_1. This allows for the smoothing of subregional ESTFRs over time, and borrowing information from countries with multiple observations and better-quality DHS estimates for countries in the subregions with fewer observations and lower quality estimates.

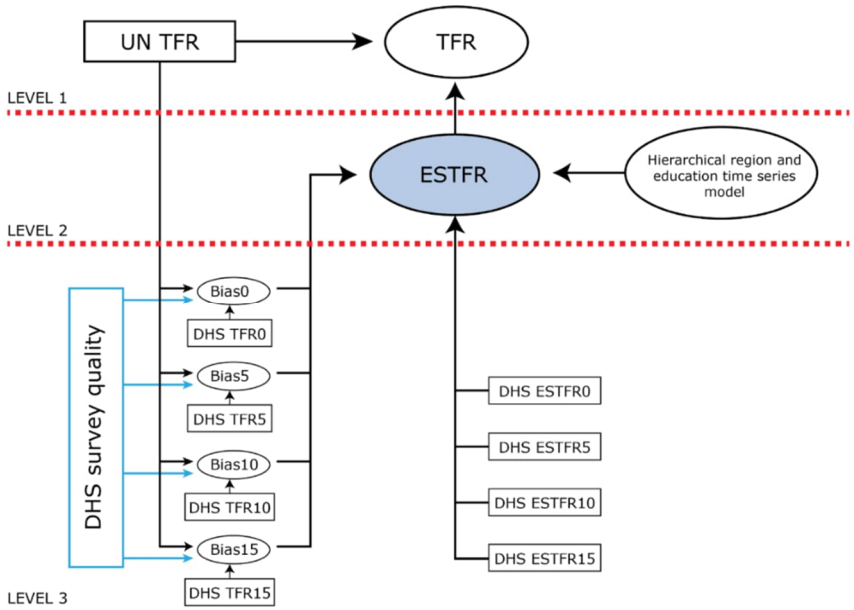
At level 3, a measurement error model corrects the biases in the country-specific ESTFRs estimated for different periods (up to 20 years preceding a survey) from the same DHS survey, denoted by $ESTFR0_{DHS}[c,e,y]$, $ESTFR5_{DHS}[c,e,y]$, and so on, as described in the Introduction and Data and Methods sections. Similar to Alkema et al. (2012), we use the UN TFR data as the unbiased reference for our TFR estimate. To our knowledge, there is no bias-corrected dataset that provides ESTFR values consistent with the UN TFR and that can be used as the ESTFR unbiased reference. Although Durowaa-Boateng, Yildiz, and Goujon (2023) also provide ESTFRs consistent with UN TFRs, they do not correct for bias in the DHS data. Furthermore, the ESTFR estimates for Latin American countries in Durowaa-Boateng, Yildiz, and Goujon (2023) are derivatives of their age- and education-specific fertility rates, for which the initial estimates were calculated using Generalized Linear Models, without correcting for biases in recall periods – as we do in this article. We constructed the ESTFR estimates for African countries using the UN-consistent model presented here.

In this article we assume that the prior distributions for the ESTFR biases reflect both the difference between country-specific DHS and UN TFRs and the quality of DHS surveys in each country and each 5-year period. Specifically, the prior distributions of the bias parameters for each quality level q, as listed in Schoumaker (2014; see Figure A-1 in Appendix A for the types of surveys used and their quality levels) ($bias0[q]$, $bias5[q]$, and so on), are centred at the difference between each repeated DHS-based TFR and the UN TFR, i.e., $TFR0[c,y]-TFR_{UN}[c,y]$, $TFR5[c,y]-TFR_{UN}[c,y]$, and so on. Each of the three described models include this bias correction by level of quality q via informative prior distributions based on the UN TFR data regardless of any adjustment to the UN TFR.

$$ESTFR0_{DHS}[c,e,y] \sim Normal(ESTFR[c,e,y] + bias0[q], \sigma_{DHS,0}). \quad (5)$$

Because we use Bayesian inference, the model needs to be completed by specifying prior distributions (or priors) and their hyperparameters for all model parameters. We assume informative priors for the above-mentioned *bias* parameters and – if such an approach is desirable – the standard deviation of the UN TFR data, σ_{UN}. All other priors are weakly informative, with low precision (i.e., inverse variance) that lets the data shape the posterior distributions of the model parameters. The full specification of all priors is provided in Appendix B (Tables B-1, B-2, B-3, and Figure B-1).

Figure 1: Model framework of reconstruction of total fertility by educational attainment



This paper discusses results obtained from three different models. The first is the UN-fully consistent model where benchmarking to the UN is achieved by setting an informative σ_{UN} parameter. The second is the UN-consistent model, referred to as the main model, where benchmarking to the UN data is data-driven, with a weakly informative prior assumed for σ_{UN}. Finally, the third model is DHS-consistent and uses the UN data for correcting the DHS biases (Equation 5) but not for benchmarking (Equation 2).

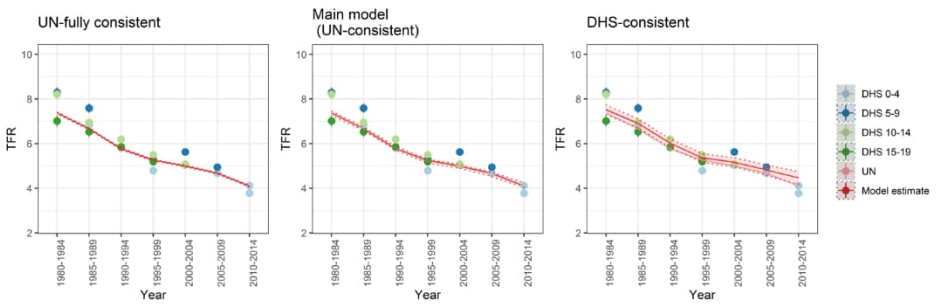
4. Results

4.1 Consistency of the model estimates with the UN WPP 2022 and DHS

To demonstrate the first step, Figure 2 shows the results for Kenya produced by the three models (estimates for all analysed countries are in the Appendix E and in Yildiz et al. 2025). The red line and the dotted red lines represent the TFR estimated by our models

and its 80% credible intervals,[7] respectively. The blue and green dots and the pink lines (which are only partly visible because of overlaps) depict TFRs estimated from the DHS using the *tfr2* Stata module (see Data and Methods section) and the UN data, respectively. In Figure 2, the left-hand panel shows the estimates from the UN-fully consistent model that are perfectly aligned with the UN data (which explains the full overlap of the UN with the red line, which obscures the pink line). The very narrow credible interval (barely visible in the plot) is a direct result of the high precision assumed for the UN TFR ($\sigma_{UN} = 1/\sqrt{1000}$ implies that the UN TFR is assumed to be within ± 0.06 interval around the estimated TFR value). In the middle panel, the UN-consistent model represents estimates from a model where benchmarking to the UN data is data-driven, with a weakly informative prior assumed for parameter σ_{UN}. Finally, the right-hand panel shows estimates from the DHS-consistent model. Similar to the UN TFR, the TFR from our three models tend to be consistently lower than the DHS TFR.

Figure 2: Comparison of TFR estimates for Kenya

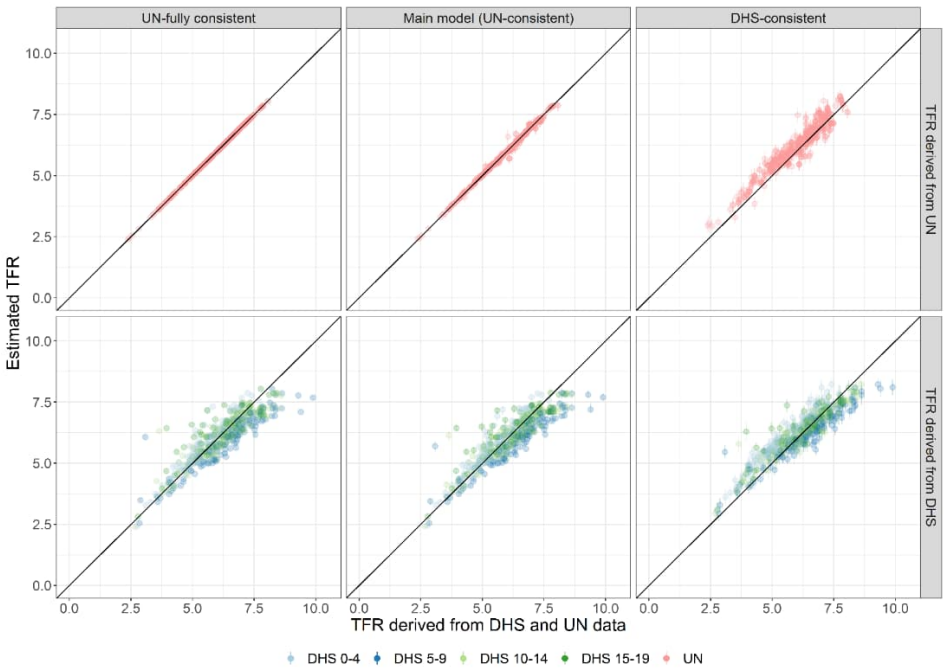


In the second step, we compare estimated fertility rates for all countries included in our study with the fertility rates reported by the UN and derived from the DHS. For Kenya, the differences between the model estimates are modest, but this is not necessarily the case for other countries, as Figure 3 shows. On the one hand, it compares estimates from our three models with the UN TFR (upper panel), and on the other hand it compares the estimates with the DHS TFR (lower panel). The panels in Figure 3 appear in the same left-to-right order as Figure 2, where the left-hand panel compares the estimated TFRs from the UN-fully consistent model, the middle panel compares the estimated TFR from the main model, and the right-hand panel compares the estimated TFR from the DHS-consistent model. Similar to Figure 2, our estimates in the left-hand panel are nearly a

[7] A credible interval is a counterpart of a confidence interval in frequentist inference. Both intervals measure the uncertainty around the 'true' parameter value. Credible interval denotes a range in which the true parameter value lies with a given probability (in our case 80% – or 0.8).

perfect match with the UN TFR (upper panel), and are centred around the DHS estimates with different recall periods (lower panel). The estimates from the main model, as shown in the middle panel (data-driven benchmarking to the UN TFR), align with the UN estimates but are slightly closer to the DHS estimates with a shorter recall period. Finally, the right-hand panel reveals that most of our estimated TFRs are higher than the UN TFRs (upper panel). This is because the estimates are not benchmarked to the UN TFR and our estimates are closer to the TFRs derived from the DHS. Our estimates are also similar to the DHS data (lower-right panel), as in the main model: that is, they fit well to the short-recall-period DHS data.

Figure 3: Estimated TFRs for all analysed countries: DHS and UN estimates compared to estimates from the three models.



Some of our estimates deviate significantly from the DHS data. For example, the UN-fully consistent model estimates the TFR for Mozambique during the 1990–1994 period as 6.07, which is similar to the UN's 6.09, while the DHS5 estimate for the same period is 3.08 (lower-left panel). Conversely, the same panel reveals a lower TFR estimate for Niger during the 2000–2004 period using the UN-fully consistent model

(7.7) compared to the DHS5 (9.9). The posterior distribution of bias parameters, which were the main drivers of the differences between our estimates and the DHS values, are available in Appendix B Figure B1. As expected, the estimated biases for poor-quality surveys and longer recall periods are higher than the biases for good-quality surveys and shorter recall periods.

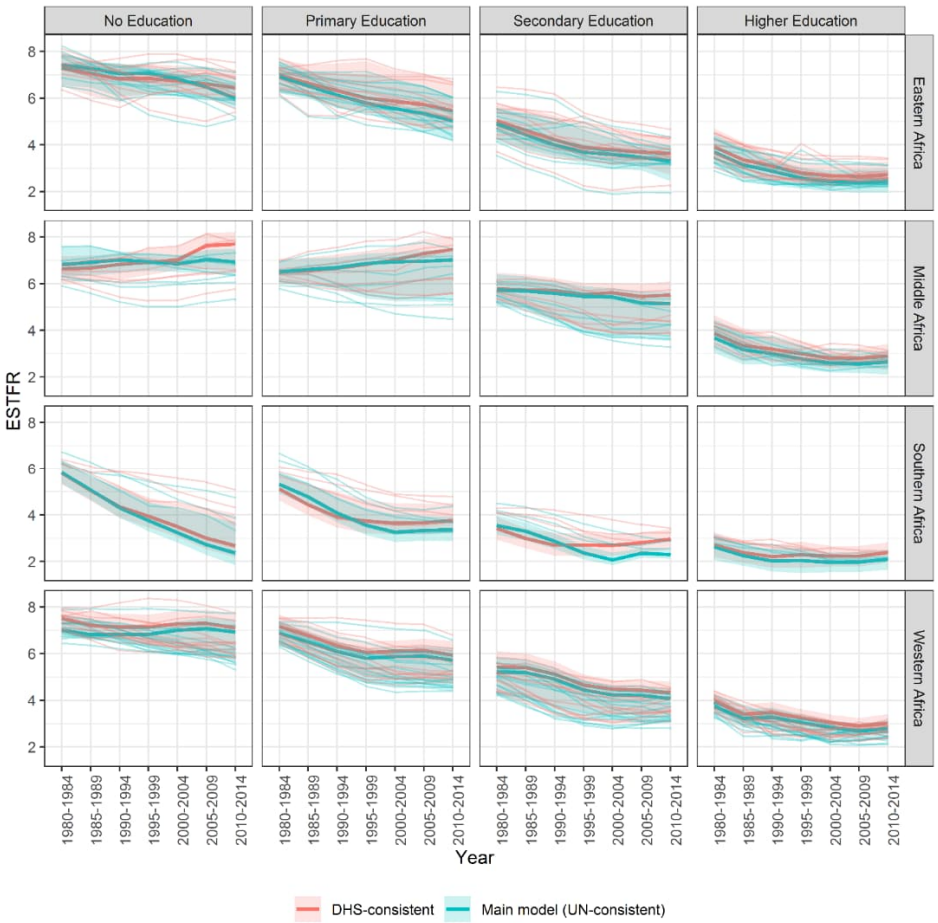
4.2 Estimated fertility rates by the level of education

In the third step, we provide the estimated education-specific total fertility rates (ESTFRs) as population-weighted averages for four sub-Saharan African regions (thick lines) and the individual country values for their respective regions (thin lines) between 1985 and 2014 (Figure 4). The estimates are obtained from the main model that ensures data-driven consistency with the UN WPP 2022 but is not identical (UN-consistent, black lines) and from the model that is consistent with the DHS (DHS-consistent, red lines). The results from the UN-fully consistent model are included in Appendix C (Figure C-3).

Overall, the fertility estimates from both models followed similar trajectories, but those from the DHS-consistent model tend to be higher than those from the UN-consistent model. This difference stems from differences in the TFR values of the DHS surveys and the UN values which the model estimates are consistent with. These differences are usually small, meaning both models' credible intervals overlap significantly and the median values fall within the 80% credible interval of the other model. Nevertheless, there are some cases where the estimates differ substantially. For instance, in Middle Africa, the DHS-consistent model estimates an increase in the fertility level of women with no and primary education and a plateau in fertility among women with secondary education in the 2000s and 2010s, whereas the UN-consistent model estimates a plateau (no and primary education) and a slight decrease (secondary education). In absolute values, the difference reaches as much as 1 child per woman among women with no education in Middle Africa. Similarly, large differences are observed among women with primary and secondary education in Southern Africa, but they follow similar trends. The differences between the two models can partly be explained by survey quality– in Middle Africa only 2 surveys out of the 18 used in the model were listed as good quality – and partially by the differences in the size of the female population in countries with different transition experiences. Both the DHS-consistent model and the main model (UN-consistent) estimate stalls in fertility decline in all educational groups in Middle and Western Africa and among women with higher education in Southern and Eastern Africa. The stalls estimated by the models have been previously reported in many studies (Goujon, Lutz, and KC 2015; Schoumaker 2009, 2019). However, as evidenced by

Kebede, Goujon, and Lutz (2019), many of these stalls are seen among women with less than higher education. The DHS-consistent related stalls that do not correspond to the UN consistent rates are not new. Indeed, Machiyama (2010) previously reviewed reports of stalls in fertility rates with DHS estimates and found varying reports of stalls in TFRs in some countries. Therefore, stalls seen in the DHS-consistent model are expected, even if this is not reflected in the UN-consistent models.

Figure 4: TFR between 1980–1985 and 2010–2015 estimated by two models, by country and region



The link between education and fertility levels is seen clearly in Figure 4. Across all the African sub-regions, women with higher education have the lowest TFRs in the years between 1980 and 2015. The hierarchy in fertility level is seen as women with no education having the highest TFR, followed by women with primary education, then women with secondary education, and finally women with higher education. Even in Middle Africa, where fertility is seemingly increasing for lower education levels (No and Primary education), women with higher education experience a consistently decreasing TFR, thus reaffirming the link between education – especially higher education – and lower fertility rates (Lutz and KC 2011).

5. Conclusions and discussion

Although there has been extensive research about the fertility–education nexus in Africa, little attention has been given to the quality of its data sources, missingness, and the inconsistencies over time and across countries (Al Zalak and Goujon 2017; Alkema et al. 2012; Schoumaker 2014). Most of the work that systematically harmonises historical and current fertility data disaggregated by education level focuses on high-income countries (e.g., Human Fertility Database, Max Planck Institute for Demographic Research, and Vienna Institute of Demography 2023) where high-quality data are plentiful and stretch far back in time. The recent effort by Durowaa-Boateng, Yildiz, and Goujon (2023) to estimate education-specific fertility rates in sub-Saharan Africa, while providing education-specific ASFRs in addition to education-specific TFRs, does not address the data quality challenges as rigorously as the methodology outlined in this article, and lacks the flexibility provided by multiple sets of estimates consistent with different data sources that meet the needs of different users. Other comparable research on low-income countries has either focused on TFRs without an educational component, or employed household surveys that place little focus on the measurement errors, completeness, and consistency of the time series. These estimates also fail to match with the UN TFR. Furthermore, population projections in the WIC Human Capital Data Explorer (WIC 2018; Lutz and KC 2011) and the work by Adhikari, Lutz, and Kebede (2024) include predicted future education-specific fertility rates, but do not provide ESTFRs for past populations, which are vital for making reasonable predictions of future ESTFRs.

To overcome the above limitations, we proposed a flexible Bayesian model to reconstruct education-specific total fertility rates (ESTFRs) from 1980–1984 to 2010–2014 in 36 sub-Saharan African countries. The proposed model combines data from two different yet imperfect sources: the UN and the DHS. The UN data are regarded as more reliable than the DHS data but are not education-specific like the DHS. To demonstrate our approach to overcoming some of the data limitations and create consistent time series

estimates, we developed three different model specifications, which allow for varying levels of consistency with the UN TFR estimates.

The DHS-consistent estimates were systematically higher than those consistent with the UN TFR. The three model estimates vary substantially in size across countries, yielding fertility trends that differ from each other – mostly in level only but sometimes in direction as well. To our knowledge, our estimates are the first education-specific fertility estimates that are consistent with the UN TFR and that fill the gaps in the time series that the DHS data suffer from.

Having access to good quality data on fertility, disaggregated by education, is essential to facilitate research on the impact of female education on fertility and to study how it differs between countries and evolves over time. Data for high-fertility countries, including those in sub-Saharan Africa, are of paramount importance, because the future fertility rates from these countries are the largest source of uncertainty regarding the size of the future global population. A key contribution of this paper is that it departs from previous work, which only provides partial information about the education-specific fertility rates needed to study the impact of education on fertility systematically over time. Likewise, with the recent pause of the DHS program, our methodology gains added relevance by providing a meticulous approach to overcoming data issues created by a lack of regular and consistent fertility surveys.

The quality issues arising from the existing fertility data have resulted in contradictory conclusions about fertility decline stalls in sub-Saharan Africa when using the same DHS data for the late 1990s and early 2000s (see Machiyama 2010 for an overview and Schoumaker 2019). While our study does not focus on the stalls, our UN-consistent estimates of fertility are systematically lower than estimates yielded by, or consistent with, the DHS. In many cases, such as our example of Kenya, stalls that are reported by the DHS data – even when cleaned and smoothed as in the *tfr2* module (Schoumaker 2013) – disappear when applying our UN-consistent estimates. The difference between the DHS(-consistent) and the UN-consistent fertility estimates is particularly large for women with lower levels of education or no education at all.

There are several differences between our work and similar research that uses Bayesian inference to quantify uncertainty. For instance, while Durowaa-Boateng, Yildiz, and Goujon (2023) employ a similar framework, there are differences in the core methodologies, particularly in error and bias handling, as discussed in previous sections. Furthermore, their work utilises previous estimates developed in this framework (Yildiz et al. 2023). This study also differs from Alkema et al. (2012), which does not disaggregate the TFR by education. First, our work covers 36 countries in Africa, compared to 7 in Alkema et al. (2012). Second, our model corrects country-specific biases in the DHS ESTFR by using UN TFR. While we achieve this correction via prior distributions that were constructed by pooling information over time, rather than by

explicitly modelling bias with survey-based covariates, our approach arguably achieves a comparable result. The results also include the uncertainty about the bias in the ESTFR estimates via the posterior distributions of the bias parameters. Third, in our models we used the varying precision values based on the recall period before the survey, which we sourced directly from the published DHS estimates (though averaged over countries). This should have moderated the heteroscedasticity effect. This is not only a different approach to the model-based approach by Alkema et al. (2012) but it is also much simpler to implement (see also Appendix B, Table B1). Finally, we relied on autoregressive rather than local smoothing over time.

Our model fit checks (Appendix C) show that the model fits the data well (Figures C-1, C-2, C-3, and C-4), with some underestimation of the observed UN TFR inputs for earlier periods observed in, e.g., Angola, Cameroon, Ethiopia, Rwanda, and Namibia. Likewise, the model fit checks show an overestimation of UN TFR for the Central African Republic, Liberia, Madagascar, Congo, and Angola – especially in the later periods (2005 to 2014). We also observed that our model tended to underestimate UN TFRs that were larger than 6.00 and overestimate those that were lower. Our results from the main model were also robust after partially removing data from the DHS.

This research relies on several assumptions and is subject to limitations. First, we used the UN TFR data to correct the biases in DHS fertility rates, which might have been subject to bias themselves. However, as discussed earlier, we believe that the errors are neither consistently too high or too low. Moreover, considering the broad use of the UN data, producing estimates consistent with them carries the potential to increase the wider use and impact of our estimates in, for example, policymaking. Second, we did not explicitly account for the modelling error in the TFR estimates produced by the Stata *tfr2* module that were used as inputs to our Bayesian model. However, the latter source of bias was most likely captured in the composite measures of bias in Equation 5. Third, the countries were grouped according to the quality of the DHS surveys as outlined in Schoumaker's (2014) quality groups. When the most recent quality level from the DHS surveys was missing for a specific country, the country was assumed to be in the same quality group as in previous surveys. Finally, we used data from 2 surveys for 6 countries (Congo Democratic Republic, Comoros, Congo, Gabon, Gambia, South Africa) and from only 1 survey for 2 countries (Central African Republic and Eswatini). The TFRs of these 8 countries were between 2.7 and 8.4 and the quality of 6 surveys was considered moderate. We acknowledge that the estimates for these countries rely more on regional trends than the other countries for which more surveys were available.

Education-specific fertility rates are required to inform policymakers and reproductive health authorities both nationally and globally. While the DHS estimates are used for analysing fertility differentials by various sociodemographic characteristics, many population projections rely on the UN national-level demographic rates. We have

developed a flexible modelling framework for different contexts and user requirements in order to address the need to fill the gap in past fertility rates disaggregated by mother's educational attainment. It allows for reconstructing the education-specific fertility rates with varying levels of consistency with the UN TFR. We found differences in past education-specific fertility rates when they were reconstructed using the three models that differ in the degree of consistency with the UN TFR. While the results from the DHS-consistent model show more frequent stalls in fertility rates for longer periods of time, the estimates from the UN-consistent model are often lower and with stalls, if present, starting later. Future users of our modelling framework and its resulting estimates can choose which set of ESTFR fits best to their specific needs.

6. Acknowledgements

The authors gratefully acknowledge funding from the Austrian Academy of Sciences' (ÖAW) Research, Science and Society Innovation Fund for the research project Bayesian Reconstruction of Populations and Vital Rates by Educational Attainment (IF_2019_29_BayesEdu).

7. Data and code availability statement

The paper employs Demographic and Health Survey data which is available for users to download after registration at https://dhsprogram.com/Data. The computer code for the work and the results are available in Zenodo repository doi:10.5281/zenodo.18185201.

References

Adhikari, S., Lutz, W., and Kebede, E. (2024). Forecasting Africa's fertility decline by female education groups. *Proceedings of the National Academy of Sciences* 121(46): e2320247121. doi:10.1073/pnas.2320247121.

Al Zalak, Z. and Goujon, A. (2017). Exploring the fertility trend in Egypt. *Demographic Research* 37(32): 995–1030. doi:10.4054/DemRes.2017.37.32.

Alkema, L., Raftery, A., Gerland, P., Clark, S.J., and Pelletier, F. (2012). Estimating trends in the total fertility rate with uncertainty using imperfect data: Examples from West Africa. *Demographic Research* 26(15): 331–362. doi:10.4054/DemRes.2012.26.15.

Amin, V. and Behrman, J.R. (2011). Do more-schooled women have fewer children and delay childbearing? Evidence from a sample of U.S. twins. *SSRN Electronic Journal*. doi:10.2139/ssrn.1968451.

Bongaarts, J. (2006). The causes of stalling fertility transitions. *Studies in Family Planning* 37(1): 1–16. doi:10.1111/j.1728-4465.2006.00079.x.

Bongaarts, J. (2008). Fertility transitions in developing countries: Progress or stagnation? *Studies in Family Planning* 39(2): 105–110. doi:10.1111/j.1728-4465.2008.00157.x.

Bongaarts, J. (2010). The causes of educational differences in fertility in Sub-Saharan Africa. *Vienna Yearbook of Population Research* 8: 31–50. doi:10.1553/populationyearbook2010s31.

Bongaarts, J. (2017), Africa's unique fertility transition. *Population and Development Review* 43(S1): 39–58. doi:10.1111/j.1728-4457.2016.00164.x.

Bongaarts, J. and Feeney, G. (1998). On the quantum and tempo of fertility. *Population and Development Review* 24(2): 271–291. doi:10.2307/2807974.

Castro Martin, T. (1995). Women's education and fertility: Results from 26 demographic and health surveys. *Studies in Family Planning* 26(4): 187–202. doi:10.2307/2137845.

Durowaa-Boateng, A., Yildiz, D., and Goujon, A.A. (2023). Bayesian model for the reconstruction of education-and age-specific fertility rates. *Demographic Research* 49(31): 809–848. doi:10.4054/DemRes.2023.49.31.

Gebreselassie, T. and Shapiro, D. (2016). *Education and fertility in Sub-Saharan Africa: What do we really know?* Paper presented at the Population Association of America Annual Meeting, Washington D.C., US.

Gietel-Basten, S. and Sobotka, T. (2021). Trends in population health and demography. *The Lancet* 398(10300): 580–581. doi:10.1016/S0140-6736(21)01051-5.

Goujon, A., Lutz, W., and KC, S. (2015). Education stalls and subsequent stalls in African fertility: A descriptive overview. *Demographic Research* 33(47): 1281–1296. doi:10.4054/DemRes.2015.33.47.

ICF (2018). Demographic and Health Surveys methodology – Standard Recode Manual for DHS-7 (DHSG4). Rockville, MD: ICF.

Jalovaara, M., Neyer, G., Andersson, G., Dahlberg, J., Dommermuth, L., Fallesen, P., and Lappegård, T. (2019). Education, gender, and cohort fertility in the Nordic Countries. *European Journal of Population* 35(3): 563–586. doi:10.1007/s10680-018-9492-2.

Kebede, E., Goujon, A., and Lutz, W. (2019). Stalls in Africa's fertility decline partly result from disruptions in female education. *Proceedings of the National Academy of Sciences* 116(8): 2891–2896. doi:10.1073/pnas.1717288116.

Liu, P. and Raftery, A.E. (2020). Accounting for uncertainty about past values in probabilistic projections of the total fertility rate for most countries. *The Annals of Applied Statistics* 14(2): 685–705. doi:10.1214/19-AOAS1294.

Lutz, W., Goujon, A., KC, S., Stonawski, M., and Stilianakis, N. (2018). *Demographic and human capital scenarios for the 21st century: 2018 assessment for 201 countries*. EU Science Hub – European Commission. https://ec.europa.eu/jrc/en/publication/demographic-and-human-capital-scenarios-21st-century-2018-assessment-201-countries.

Lutz, W. and KC, S. (2010). Dimensions of global population projections: What do we know about future population trends and structures? *Philosophical Transactions of the Royal Society B: Biological Sciences* 365(1554): 2779–2791. doi:10.1098/rstb.2010.0133.

Lutz, W. and KC, S. (2011). Global human capital: Integrating education and population. *Science* 333(6042): 587–592. doi:10.1126/science.1206964.

Machiyama, K. (2010). A re-examination of recent fertility declines in sub-Saharan Africa. (DHS Working Papers 68). Fairfax/Reston, VA: ICF Macro. http://www.measuredhs.com/pubs/pdf/WP68/WP68.pdf.

Martin, T.C. (1995). Women's education and fertility: results from 26 Demographic and Health Surveys. *Studies in Family Planning* 26(4): 187–202. doi:10.2307/2137845.

Max Planck Institute for Demographic Research and Vienna Institute of Demography (2023). Human Fertility Database [Data set]. Rostock and Vienna: MPIDR and VID. www.humanfertility.org.

May, J.F. and Rotenberg, S. (2020). A call for better integrated policies to accelerate the fertility decline in sub-Saharan Africa. *Studies in Family Planning* 51(2): 193–204. doi:10.1111/sifp.12118.

Rajaratnam, J.K., Marcus, J.R., Flaxman, A.D., Wang, H., Levin-Rector, A., Dwyer, L., Costa, M., Lopez, A.D., and Murray, C.J. (2010). Neonatal, postneonatal, childhood, and under-5 mortality for 187 countries, 1970–2010: A systematic analysis of progress towards Millennium Development Goal 4. *The Lancet* 375(9730): 1988–2008. doi:10.1016/S0140-6736(10)60703-9.

Ryder, N.B. (1986). The cohort as a concept in the study of social change. In: Mason, W.M. and Fienberg, S.E. (eds). *Cohort analysis in social research: Beyond the identification problem*. New York, NY: Springer: 9–44. doi:10.1007/978-1-4613-8536-3_2.

Schoumaker, B. (2008). *Stalls and reversals in fertility transitions in sub-Saharan Africa: Real or spurious?* Paper presented at the IUSSP Seminar Human Fertility in Africa: Trends in the last decade and prospects for change, September 16–18, 2008 in Cape Coast, Ghana.

Schoumaker, B. (2009). Stalls in fertility transitions in sub-Saharan Africa: Real or spurious? (Document de travail SPED 30.) Louvain-la-Neuve: Département des sciences de la population et du développement, Université Catholique de Louvain.

Schoumaker, B. (2010). *Reconstructing fertility trends in sub-Saharan Africa by combining multiple surveys affected by data quality problems*. Paper presented at the 2010 Annual Meeting of the Population Association of America, April 15–17, 2010 in Dallas, TX.

Schoumaker, B. (2011). *Omissions of births in DHS birth histories in sub-Saharan Africa: Measurement and determinants*. Paper presented at the 2010 Annual Meeting of the Population Association of America, March 31–April 2, 2011 in Washington, D.C.

Schoumaker, B. (2013). A Stata module for computing fertility rates and TFRs from birth histories: tfr2. *Demographic Research* 28(38): 1093–1144. doi:10.4054/DemRes.2013.28.38.

Schoumaker, B. (2014). Quality and consistency of DHS fertility estimates, 1990 to 2012. (DHS Methodological Reports 12). Washington, D.C.: ICF International.

Schoumaker, B. (2019). Stalls in fertility transitions in sub-Saharan Africa: Revisiting the evidence. *Studies in Family Planning* 50(3): 257–278. doi:10.1111/sifp.12098.

Schoumaker, B.D. and Sánchez-Páez, D.A. (2024), Disruptions in educational progress and fertility dynamics by educational level: Unraveling the link between education and fertility stalls in sub-Saharan Africa. *Population and Development Review* 50(1): 59–85. doi:10.1111/padr.12610.

Schultz, T.P. (1997). Demand for children in low income countries. In: Rosenzweig, M.R. and Stark, O. (eds.). *Handbook of population and family economics.* Amsterdam: Elsevier: 349–430. doi:10.1016/S1574-003X(97)80025-6.

Ševčíková, H., Alkema, L., and Raftery, A.E. (2011). bayesTFR: An R package for probabilistic projections of the total fertility rate. *Journal of Statistical Software* 43(1): 1–29. doi:10.18637/jss.v043.i01.

Shapiro, D. (2012). Women's education and fertility transition in sub-Saharan Africa. *Vienna Yearbook of Population Research* 10(1): 9–30. doi:10.1553/populationyearbook2012s9.

Shapiro, D. and Gebreselassie, T. (2008). Fertility transition in sub-Saharan Africa: Falling and stalling. *African Population Studies* 23(1): 3–23. doi:10.11564/23-1-310.

Skirbekk, V. (2008). Fertility trends by social status. *Demographic Research* 18(5): 145–180. doi:10.4054/DemRes.2008.18.5.

Sneeringer, S.E. (2009). Fertility transition in sub-Saharan Africa: A comparative analysis of cohort trends in 30 countries. (DHS Comparative Reports 23). Calverton, MD: DHS.

United Nations (2022). *World population prospects: The 2022 revision*. New York City: Department of Economic and Social Affairs, Population Division. http://esa.un.org/unpd/wpp/.

United Nations, Department of Economic and Social Affairs, Population Division (2022). World Population Prospects 2022: Methodology of the United Nations population estimates and projections. (UN DESA/POP/2022/TR/NO. 4). New York City: UNDESA.

USAID (2023). *The DHS Program: Demographic and Health Surveys*. Washington, D.C.: ICF International. https://dhsprogram.com/.

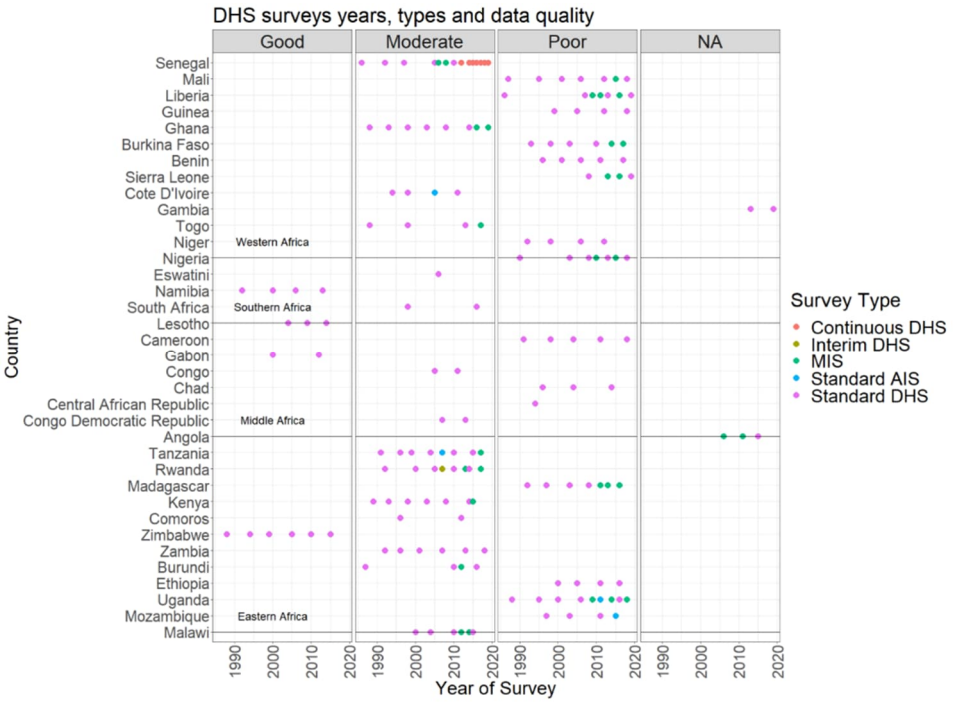
WIC – Wittgenstein Centre for Demography and Global Human Capital (2018). Wittgenstein Centre Data Explorer Version 2.0. Available at: http://www.wittgensteincentre.org/dataexplorer.

Yildiz, D., Wiśniowski, A., Brzozowska, Z., and Durowaa-Boateng, A.A. (2023). A flexible model to reconstruct education-specific fertility rates: Sub-Saharan Africa case. (Vienna Institute of Demography Working Papers). doi:10.1553/0x003e65e0.

Yildiz, D., Wiśniowski, A., Brzozowska, Z., and Durowaa-Boateng, A.A. (2025). A flexible model to reconstruct education-specific fertility rates: Sub-Saharan Africa case study version 2 (Zenodo, November 2025). doi:10.5281/zenodo.18185201.

Appendix A – DHS surveys and their characteristics

Figure A-1: Types of surveys used in the research and their characteristics



Source: The quality assessment of DHS surveys is based on Schoumaker (2014).

Appendix B – Prior distributions for model parameters

Prior distributions (or priors)[8] for the autoregressive model parameters, β_0 and β_1, are centred at the observed mean of the DHS ESTFR values for the period 1980–1984 and the remaining periods after 1980–1984, respectively (denoted below by $b0$ and $b1$). These values are displayed in Table B1. We also assume that the time series

[8] In Bayesian inference, each estimable parameter requires a distribution that reflects the modeller's knowledge of it before seeing the data. Often, we use weakly informative priors that are convenient for the numerical stability of the estimation algorithms and 'let the data speak for themselves', i.e., show a strong preference for the signal in the data rather than the prior itself.

autoregressive models are stationary (parameter β_2 is within a (0,1) interval with a weakly informative prior):

$$\begin{gathered}\beta_0[1,r,e] \sim Normal(b0, \tau_{b0}) \\ \beta_1[1,r,e] \sim Normal(b1, \tau_{b1}) \\ \beta_2[1,r,e] \sim Normal\ (0.5,1), \quad \beta_2[1,r,e] \in (0,1)\end{gathered}$$

The JAGS software, which was used for the Bayesian hierarchical model, requires precision parameters as an input instead of standard deviation. Hence, we use τ for the precision and σ for the standard deviation in the specifications for the priors shown below.

$$\begin{gathered}\tau_{b0} = 1/\sigma_{b0}^2 \\ \tau_{b1} = 1/\sigma_{b1}^2 \\ \sigma_{b0} \sim Normal(1.365, \quad b0_{prec}) \\ \sigma_{b1} \sim Normal(1.543, \quad b0_{prec})\end{gathered}$$

The observed standard deviations in DHS ESTFR for the period 1980–1984 and the remaining periods after 1980–1984 are 1.365 and 1.543, respectively.

Precision (inverse variance) is

$$\begin{gathered}b0_{prec} \sim Uniform(0,1) \\ 1/\sigma^2{}_{10} \sim Normal_+(5,10) \\ 1/\sigma^2{}_{11} \sim Normal_+(5,10),\end{gathered}$$

where $Normal_+$ denotes a positive, half-normal distribution. In the main model, σ_{UN}[c,y] follows a country- and year-specific half-normal distribution with a mean equal to 1% of the TFR [c,y].

$$\sigma_{UN}[c,y,] \sim Normal_+(TFR * 0.01, 100).$$

This prior provides sufficient range for the variability of the UN TFR without causing numerical instability and is, practically, very weakly informative. The range of posterior distribution of $1/(\sigma_{UN}[c,y])^2$ is contained within 0.0 and 0.4.

The prior for $\sigma_{UN}[c,y]$ is not specified in the model without UN data; it is fixed at σ_{UN}= 0.03 in the UN-fully consistent model.

$$\begin{gathered}1/(\sigma_{DHS,x})^2 \sim Normal(1/(d_x)^2, d_{prec}) \\ d_{prec} \sim Uniform(0,1).\end{gathered}$$

The expected value for the precision of the DHS data on ESTFR (d_x) is sourced directly from the relevant DHS data. The values used in the estimation are available in Table B-2.

Table B-1: Prior values for hyperparameters $b0$ and $b1$

Subregion	Education	Year	$b0$	$b1$
Eastern Africa	No Education	1980–1984	7.32	6.99
	Primary Education	1980–1984	6.99	6.37
	Secondary Education	1980–1984	5.40	4.27
	Higher Education	1980–1984	4.19	3.04
Middle Africa	No Education	1980–1984	6.89	6.79
	Primary Education	1980–1984	6.64	6.32
	Secondary Education	1980–1984	5.69	4.95
	Higher Education	1980–1984	4.32	3.30
Southern Africa	No Education	1980–1984	6.21	4.61
	Primary Education	1980–1984	5.63	4.47
	Secondary Education	1980–1984	3.84	3.16
	Higher Education	1980—1984	3.05	2.60
Western Africa	No Education	1980–1984	7.42	7.26
	Primary Education	1980–1984	7.06	6.12
	Secondary Education	1980–1984	5.13	4.58
	Higher Education	1980–1984	4.57	3.70

Source: Authors' own calculation using ESTFR estimated by the tfr2 module

Table B-2: Prior values for hyperparameter d_x (rounded)

Period	dx_
0	1.631
5	1.948
10	1.720
15	1.529

Source: Authors' own calculation using ESTFR estimated by the tfr2 module

Priors for the period- and quality-group-specific bias parameters and associated precisions are calculated by subtracting the UN TFR from the DHS TFR estimated by the *tfr2* module for different recall periods.

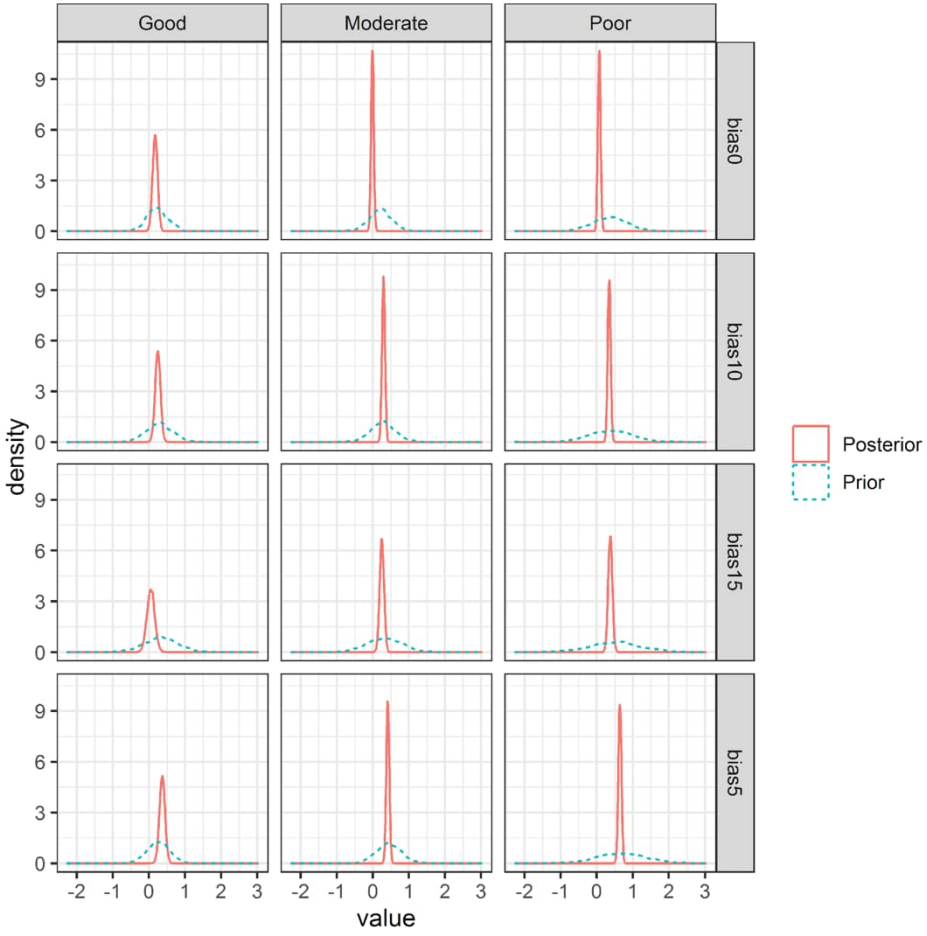
$$bias[q,p] \sim Normal\ (biasmean[q,p]\ , biasprec[q,p])$$

Table B-3: Priors for the mean and the precision for the bias parameters

Period	Quality	Mean (biasmean)	Precision (biasprec)
0	Poor	0.364	4.291
0	Moderate	0.223	9.746
0	Good	0.218	12.019
5	Poor	0.723	2.233
5	Moderate	0.449	8.874
5	Good	0.241	9.860
10	Poor	0.393	3.156
10	Moderate	0.291	8.548
10	Good	0.277	7.492
15	Poor	0.523	1.981
15	Moderate	0.329	4.338
15	Good	0.308	4.382

Source: Authors' own calculation using ESTFR estimated by the tfr2 module and the UN TFR.

Figure B-1: Prior and posterior distributions of bias parameters based on years before the survey and the quality of the DHS survey



Appendix C – Posterior predictive checks

We predicted the UN TFR using the main model (UN-consistent) as shown in Figure C1 and Figure C2, which show a comparison between predicted UN TFRs and actual UN TFRs by year, and by country and year. These are based on the posterior predictive distribution of the UN TFR data used as input to the model. We observe that the model generally predicts the patterns correctly. Notable exceptions include underestimation for Namibia (1985–1989), Mozambique (1985–1994), and Rwanda (1980–1989), although predictive intervals contain the data points. The model overestimates the UN TFR (again, predictive intervals contain data) for Angola (2005–2014) and Uganda (1995–1999).

Figure C-1: Comparison of predicted UN TFR and UN TFR

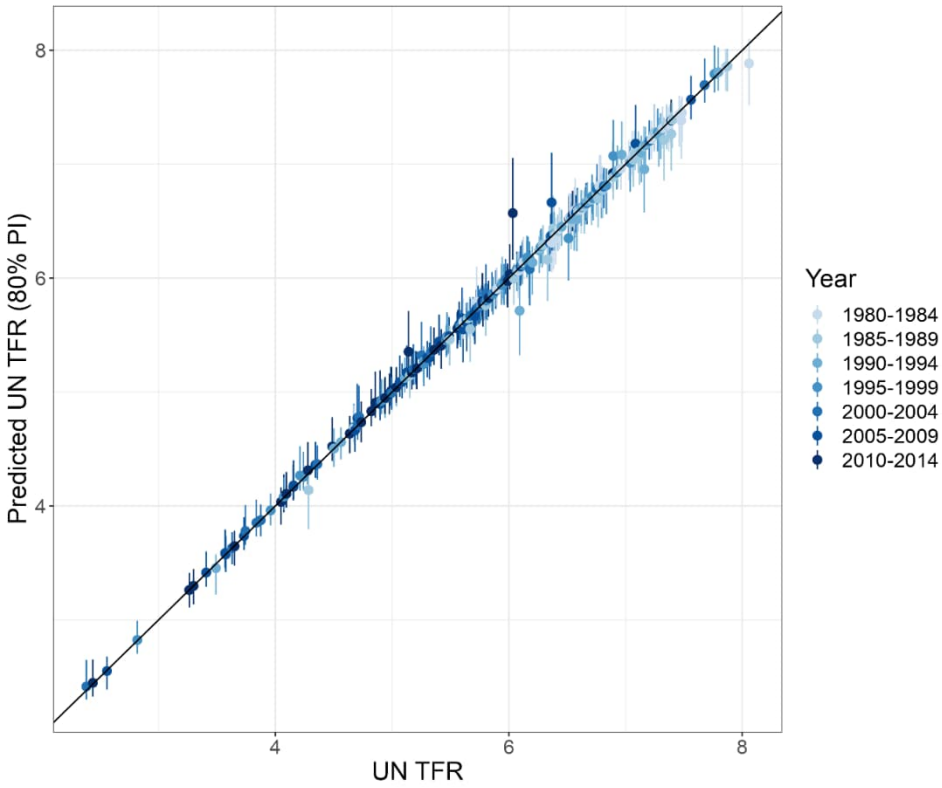
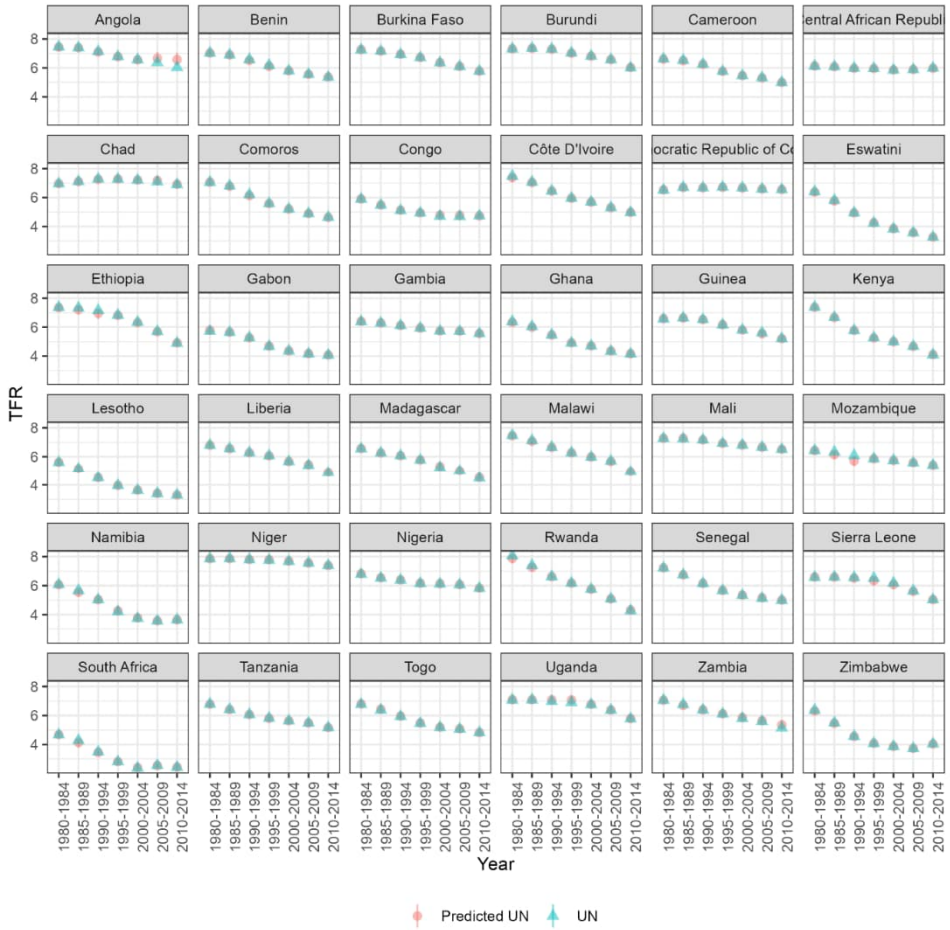


Figure C-2: Comparison of predicted UN TFR and UN TFR by country and year



We also estimated ESTFRs by separately removing ESTFR0, ESTFR5, ESTFR10, and ESTFR15 from the inputs and compared the resulting ESTFRs to those from the main model (UN-consistent) in Figure C3. Finally, we predicted ESTFR0, ESTFR5, ESTFR10 and ESTFR15 in the main model and compared them to the ESTFRs derived from the DHS birth histories in Figure C4.

Figure C-3: Comparison of the results of the Main model (UN-consistent) with partially removed data along with their 80% confidence intervals

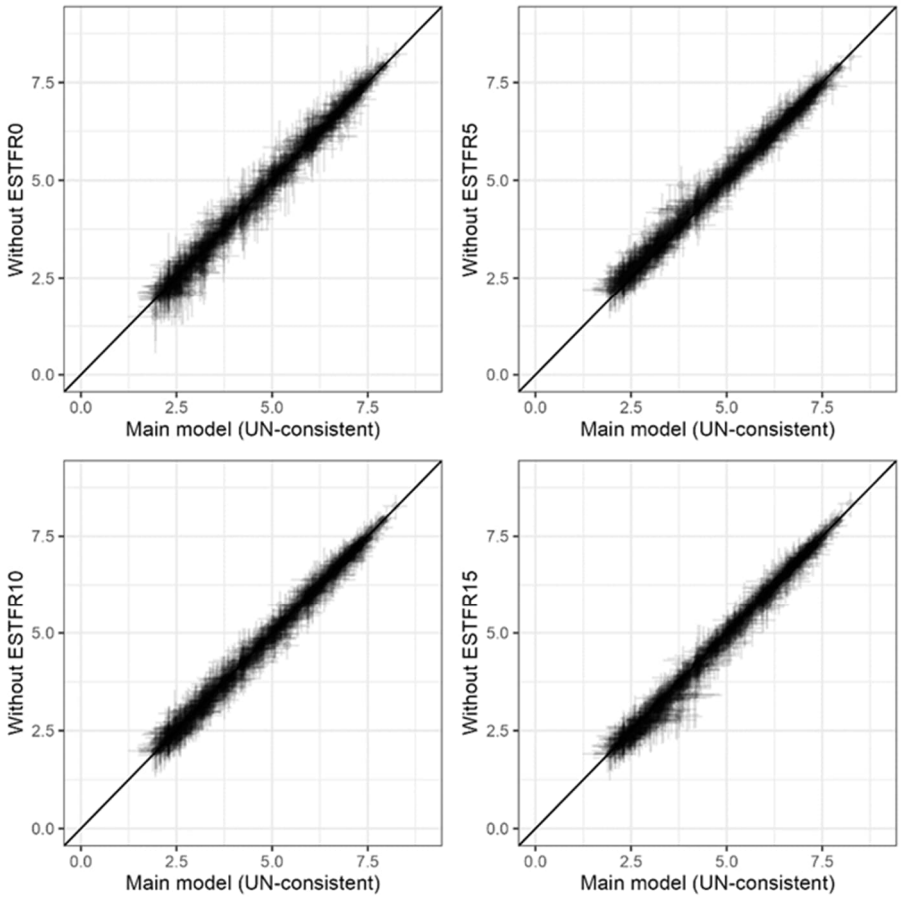
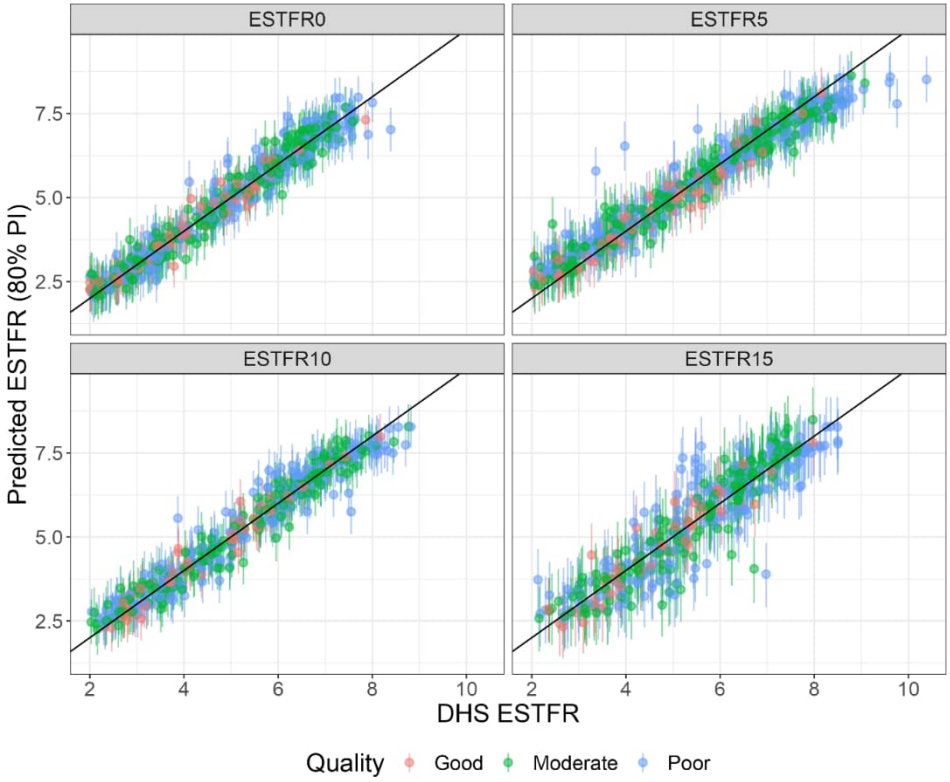


Figure C-4: Comparison of predicted ESTFR and DHS ESTFR by recall period and quality group along with their 80% confidence intervals



Appendix D – Abbreviations

Table D-1: List of abbreviations and descriptions

Abbreviation	Description
AIS	AIDS Indicator Survey
DHS	Demographic and Health Surveys
ESTFR	Education-Specific Fertility Rate
MIS	Malaria Indicator Survey
TFR	Total Fertility Rate
UN	United Nations
USAID	United States Agency for International Development
WIC	Wittgenstein Centre for Demography and Global Human Capital
WPP	World Population Prospects